The World of Quantum Matter

ALBERT-LUDWIGS-UNIVERSITÄT FREIBURG

Atomic and Molecular Quantum Dynamics

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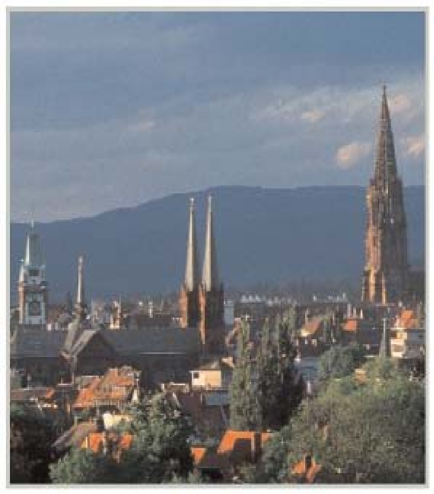
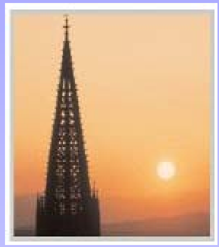
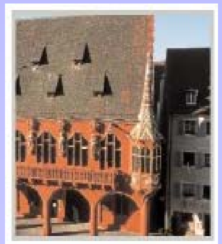
Albert-Ludwigs-Universität Freiburg

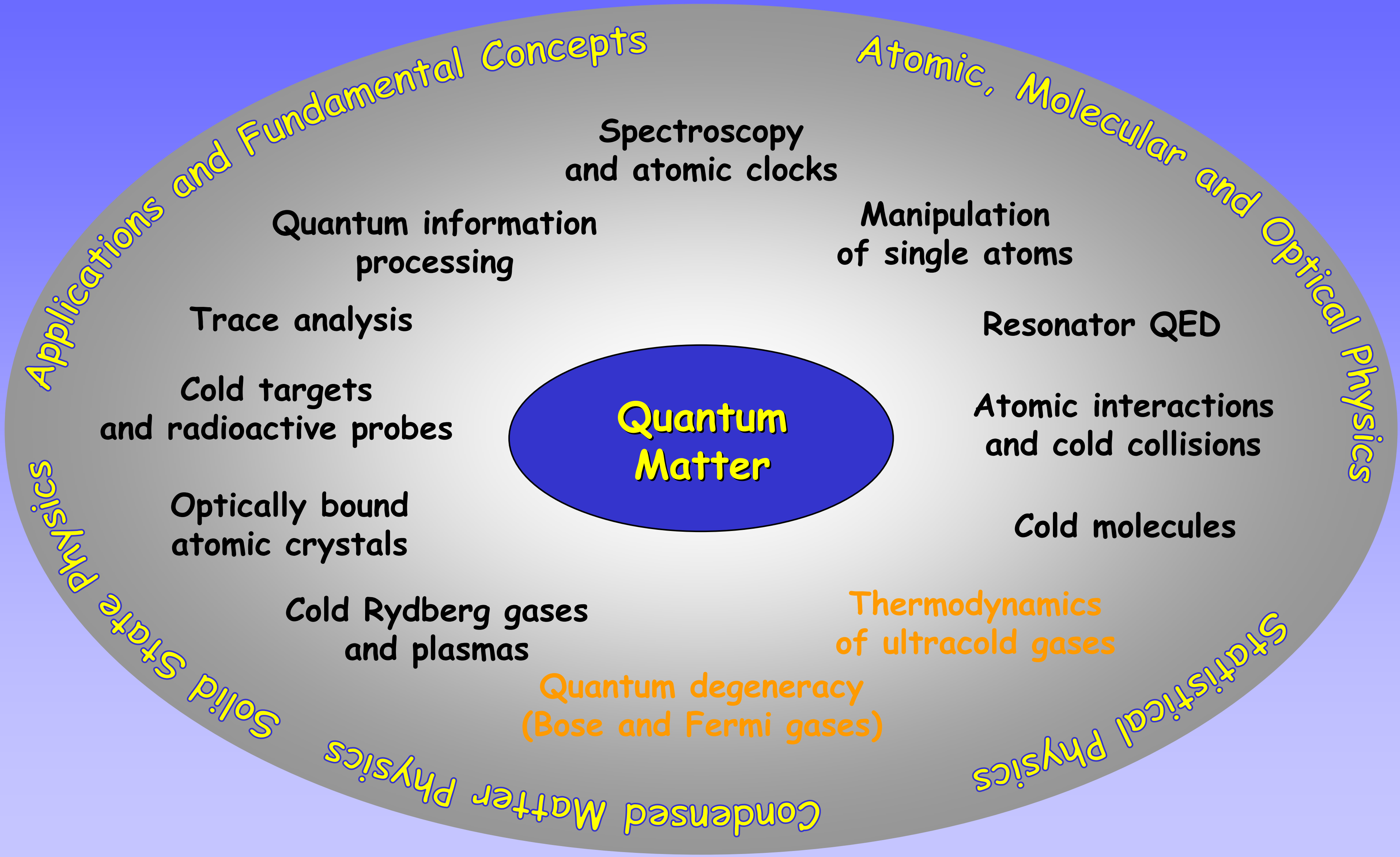
Atomare und Molekulare Quantendynamik







The World of Quantum Matter



Contents of the lectures

0. Primer on light-matter interactions
1. The way to absolute zero –
   cooling and trapping methods for atoms

Lecture 1

2. Cold collisions
3. **Bose-Einstein condensation**
4. **Degenerate Fermi gases**

Lecture 2

5. Cold Rydberg gases and plasmas
6. Ultracold molecules

Lecture 3

7. Manipulation of single atoms
8. Cold atoms as targets for photon and particle beams

Lecture 4

Bosons and Fermions

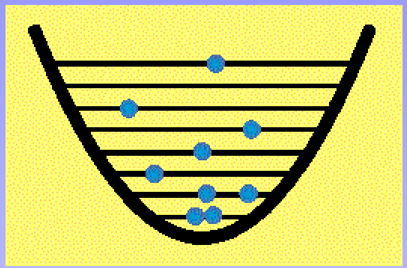
Quantum particles appear in two different “flavors” (quantum statistics) depending on their total angular momentum (spin):

integer spin	**Bosons** (e.g. photon)
half-integer spin	**Fermions** (e.g. electron, proton, neutron)

The spin determines the social quantum behaviour of these particles:

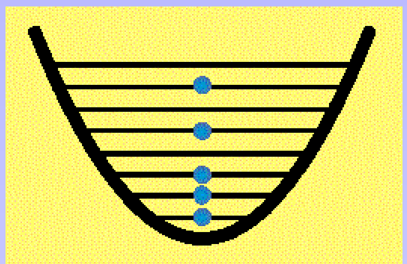
Bosons:

A quantum state can be occupied by an arbitrary number of bosons. If the state is already occupied by N bosons, the probability for the next boson to occupy the same state is N times enhanced.

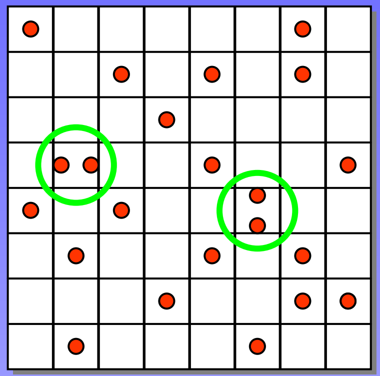


Fermions:

A quantum state can never be occupied by more than one fermion.



Quantum statistics



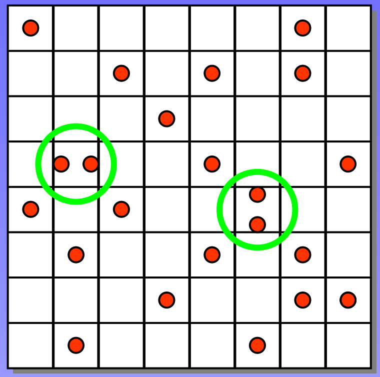
classical particles

$$p_i = \frac{1}{N}$$

occupation per state
follows simple Poissonian statistics

courtesy Rudi Grimm (Universität Innsbruck)

Quantum statistics



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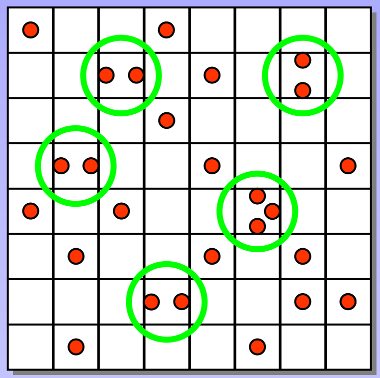


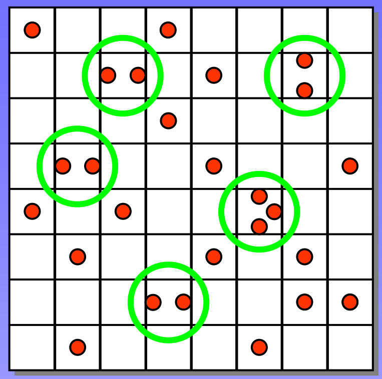
bosons

$$p_i \propto n_i + 1, \qquad n_i = 0, 1, 2, \ldots$$

bunching effect
(well known Hanbury-Brown-Twiss expt.)

courtesy Rudi Grimm (Universität Innsbruck)

Quantum statistics



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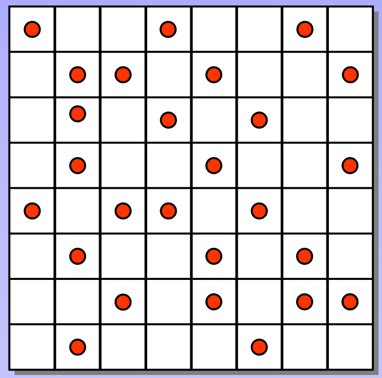


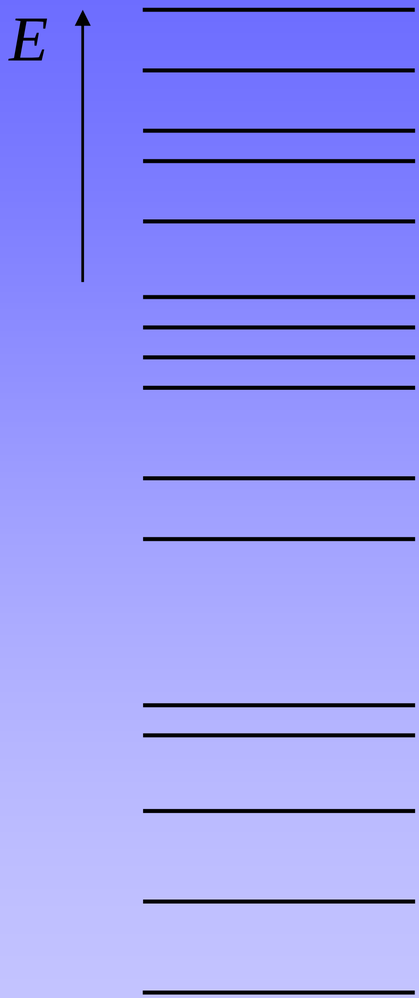
fermions

$$p_i \propto n_i - 1, \qquad n_i = 0, 1$$

Pauli's exclusion principle

courtesy Rudi Grimm (Universität Innsbruck)

Boltzmann factor



quantum states with different energies

$$p \propto \mathrm{e}^{-E/k_{\mathrm{B}}T}$$

Boltzmann factor

Boltzmann´s constant
k_B = 1.3805×10^{-23} J/K

Ludwig Boltzmann

courtesy Rudi Grimm (Universität Innsbruck)

Distribution functions

Bose-Einstein statistics

$$f_{\mathrm{BE}} = \frac{1}{\mathrm{e}^{(E-\mu)/k_{\mathrm{B}}T} - 1}$$

Boltzmann factor
together with
quantum statistics

$$f_{\mathrm{cl}} = \frac{1}{\mathrm{e}^{(E-\mu)/k_{\mathrm{B}}T}}$$

classical limit

$$f_{\mathrm{FD}} = \frac{1}{\mathrm{e}^{(E-\mu)/k_{\mathrm{B}}T} + 1}$$

Fermi-Dirac statistics

courtesy Rudi Grimm (Universität Innsbruck)

Thermodynamics

High temperature regime ($k_B T \gg \Delta E_{q.m.}$):

Each quantum mechanical state is occupied with a probability $\ll 1$
$\Rightarrow$ no difference between bosons and fermions (**Boltzmann statistics**)

Low temperature regime ($k_B T \lesssim \Delta E_{q.m.}$):

Each quantum mechanical state is occupied with a probability $\gtrsim 1$
$\Rightarrow$ **Bose-Einstein statistics** and **Fermi-Dirac statistics**

Level spacing $\Delta E_{q.m.}$ for particles with an average spacing d:

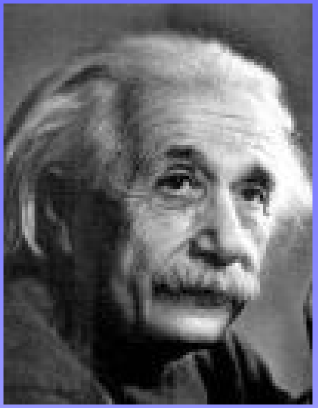
$$\Delta E_{q.m.} \sim p^2 / 2m \sim h^2 / 2md^2$$

$$k_B T \lesssim \Delta E_{q.m.} \quad \Leftrightarrow \quad \boxed{\mathbf{n \Lambda_{dB}^3 \gtrsim 1}}$$

thermal deBroglie wavelength $\Lambda_{dB} = (2\pi\hbar^2 / mk_B T)^{1/2}$

Prediction in 1925

Satyendranath Bose
(1894 - 1974)

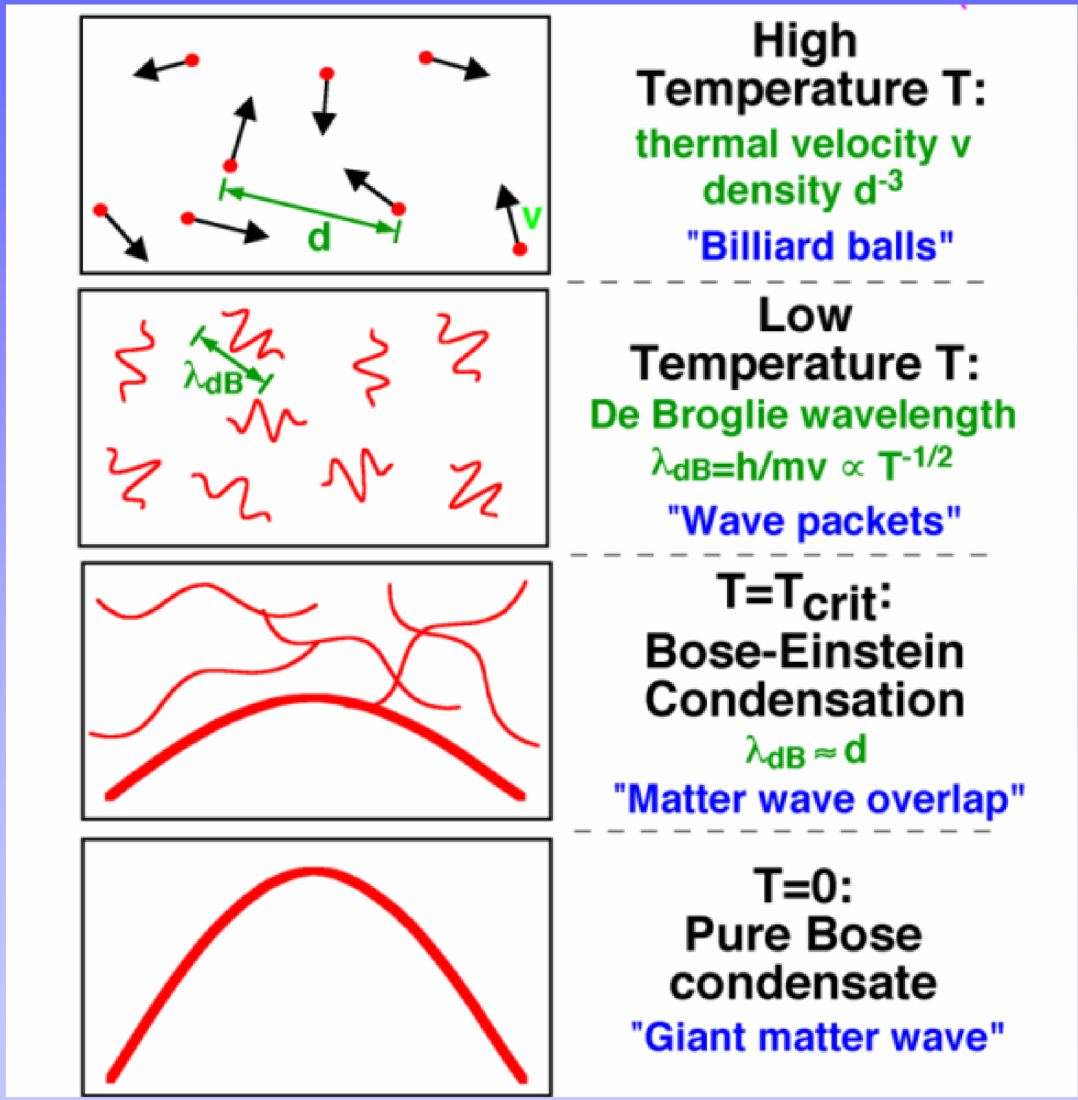


Albert Einstein
(1879 - 1955)

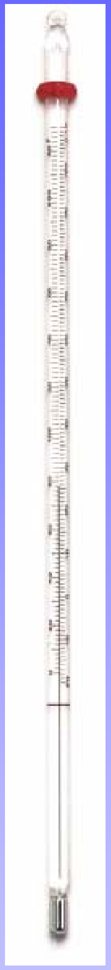
An **“ideal” gas** of **Bosons** shows a **phase transition** at sufficiently **low temperatures.**

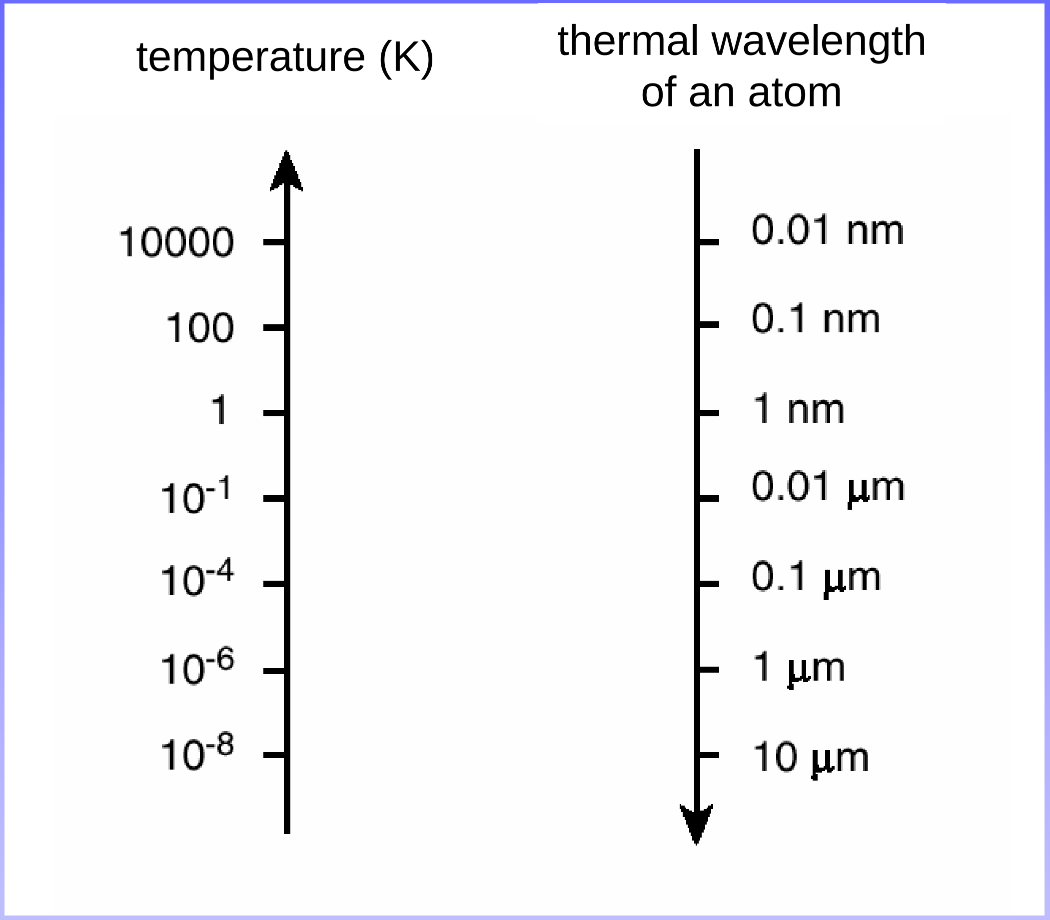
The gas condenses into the lowest available quantum state and forms a **macroscopic quantum object**:
the **Bose-Einstein condensate**.

Bose-Einstein condensation (BEC)



Thermal deBroglie wavelength

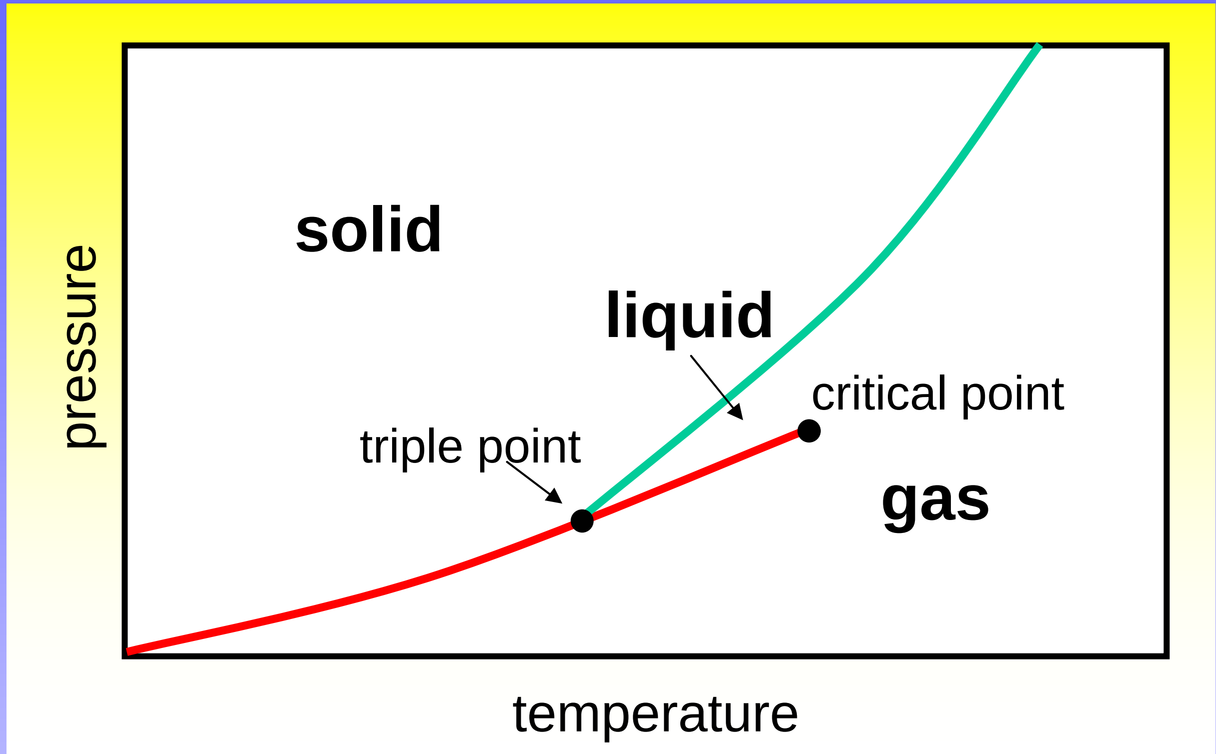




At room temperature (300 K):
Bose condensation would occur at a density larger than $(0.1\ \text{nm})^{-3} = 10^{24}$ atoms/cm^3
(typical density in a solid sample !)

Why we do not observe Bose condensation in real life ?

Phase diagram



At low temperatures, the thermal equilibrium state of every systemis the **solid phase** (even down to T = 0)

Bose-Einstein condensation of a gas can only occur as a **metastable phase** at low densities ($\sim 10^{14}$ atoms/cm^3) to prevent 3-body-recombination (equivalent to a saturated vapor)

$\Rightarrow$ **ultralow temperatures required** ($\sim$ 1 μK)

Are atoms bosons?

Atoms and molecules are composite particles, composed of fermionic elementary particles (electrons, protons, neutrons).

Total spin is integer (total number of electrons, protons and neutron is even)
⇒ atoms and molecules are ***bosonic***

Total spin is half-integer (total number of electrons, protons and neutron is odd)
⇒ atoms and molecules are ***fermionic***

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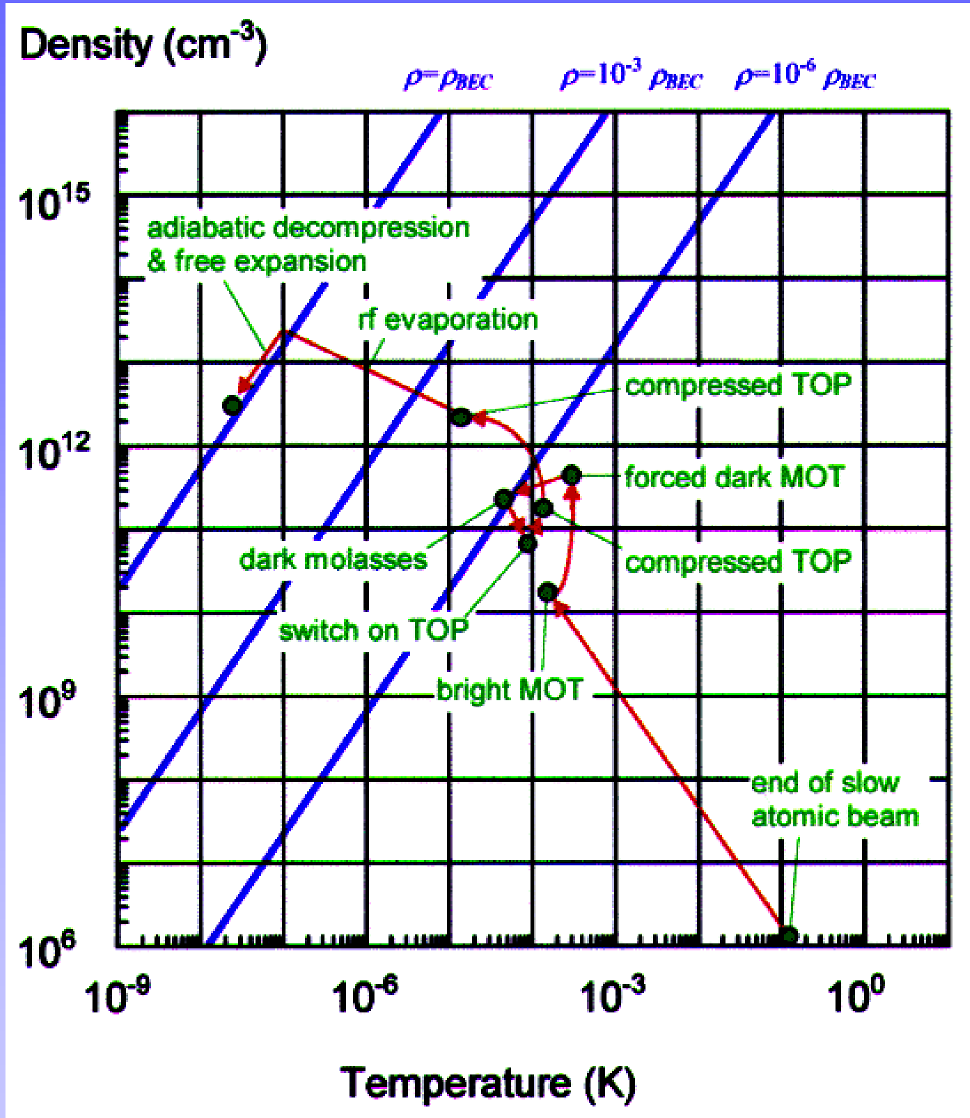
$\Rightarrow$ atoms and molecules are ***fermionic***

Under which conditions is a composite particle a composite particle?

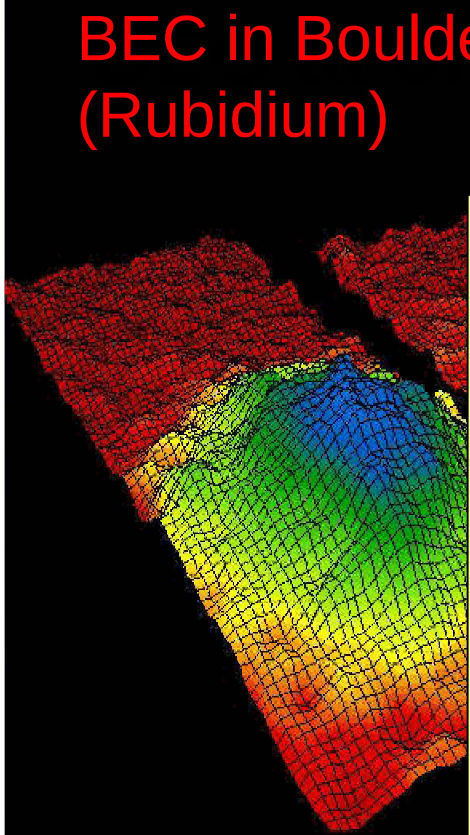
Thermal energy $k_B T$ smaller than the internal excitation of the particle
$\Rightarrow$ internal degrees of freedom are frozen out and do not matter for the thermodynamics

Interaction energy smaller than the internal excitation energy
$\Rightarrow$ collisions do not remove or excite the bound electrons

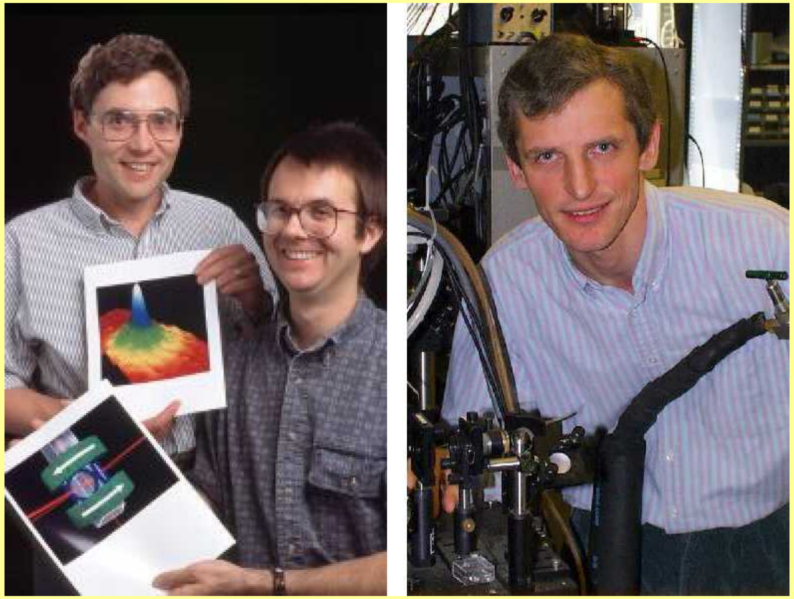
The long, long road to BEC



BEC in Boulder, Juni 1995
(Rubidium)



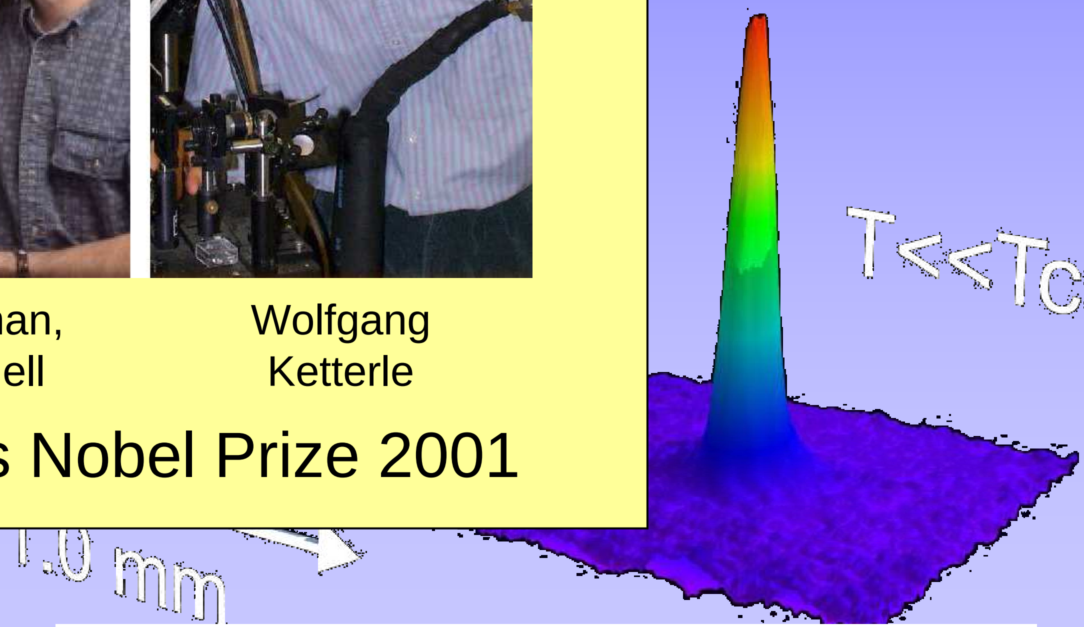
Bose-Einstein condensation



Carl Wieman,
Eric Cornell

Wolfgang
Ketterle

Physics Nobel Prize 2001

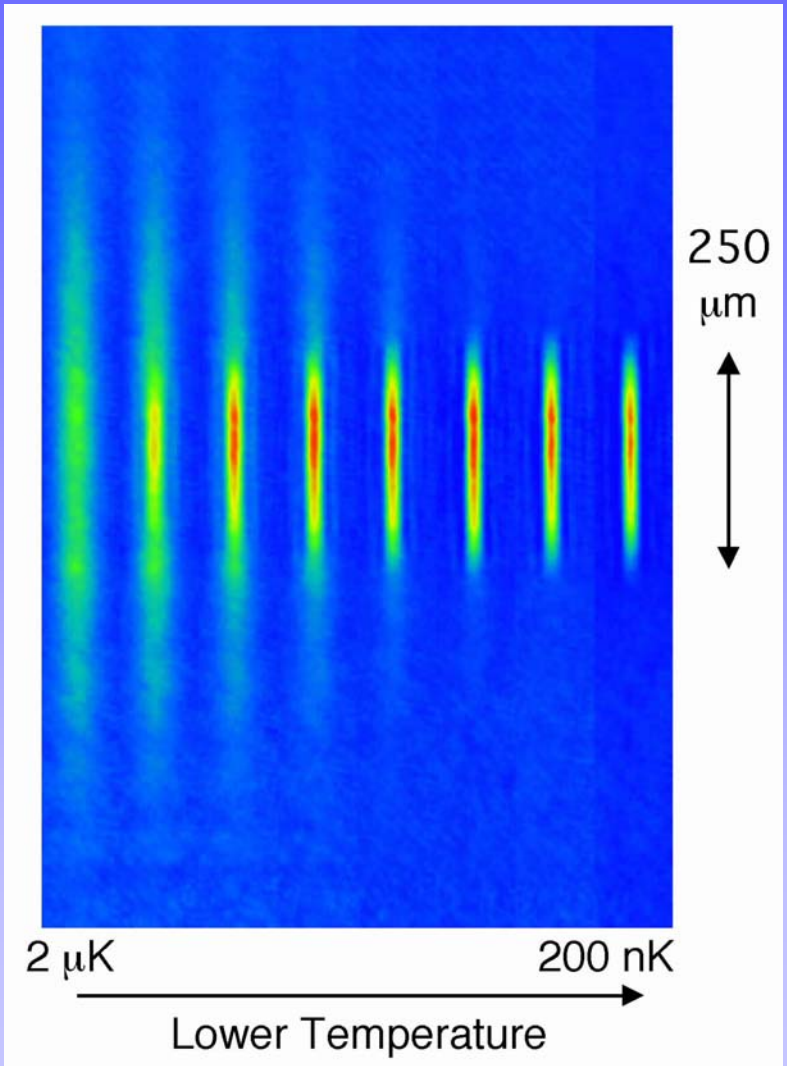


BEC at MIT, Nov. 1995 (Natrium)

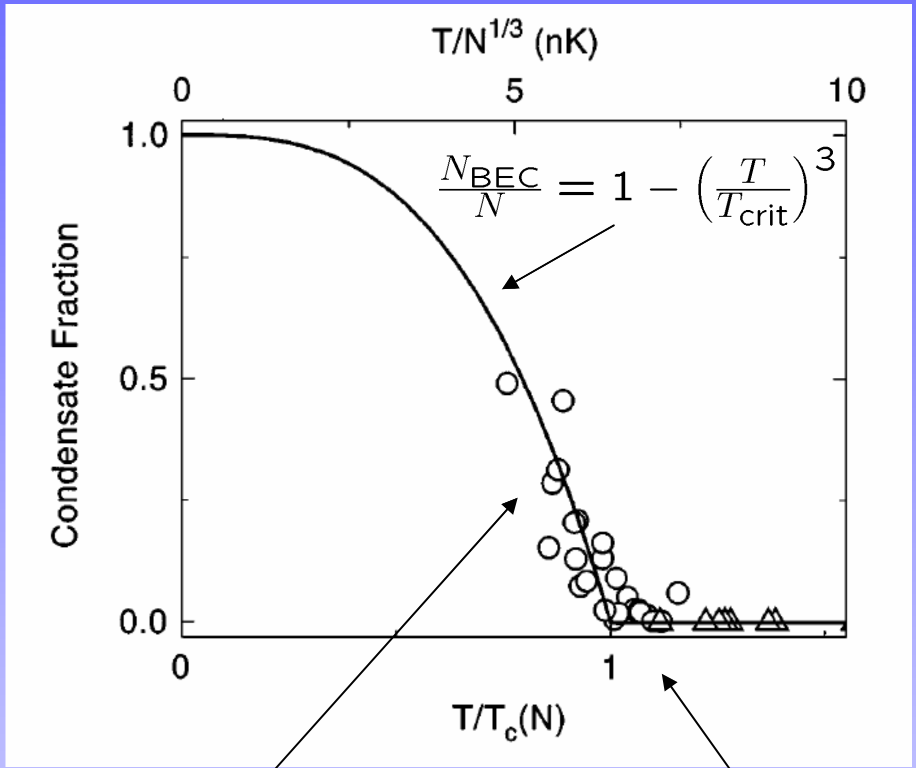
BEC - “molecule” of the year

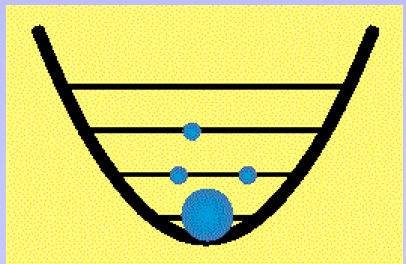
Transition to BEC

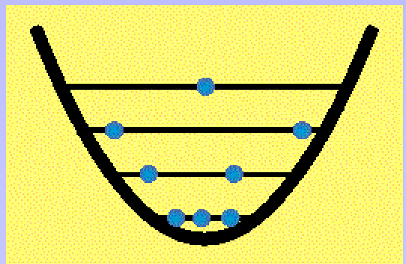
In-situ measurement



Number of atoms in the condensate





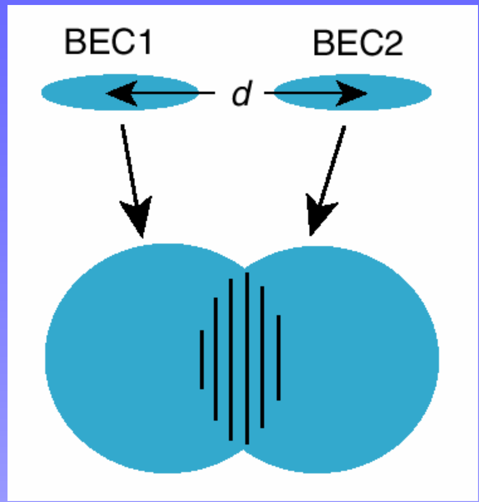


W. Ketterle *et al.* @ MIT

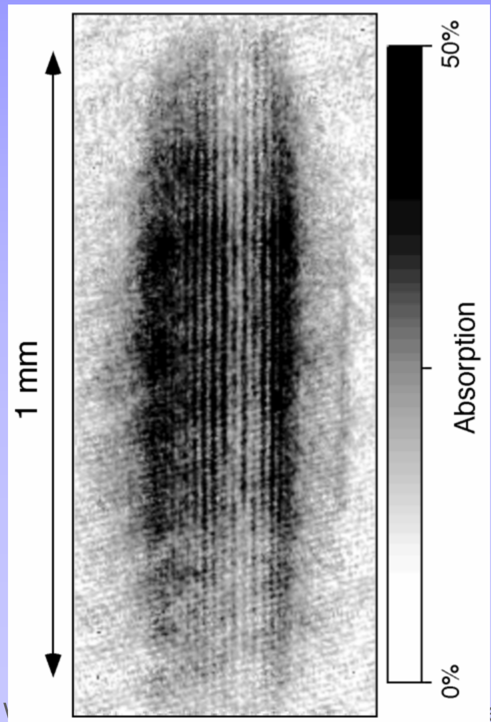
Matter-wave interference

trapped BECs

BECs after expansion
(expansion time Δt)

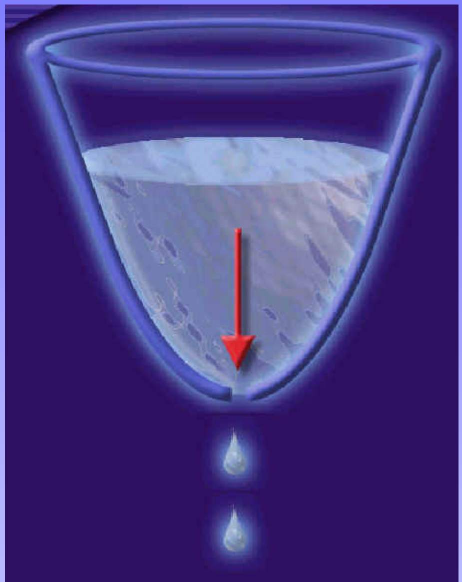


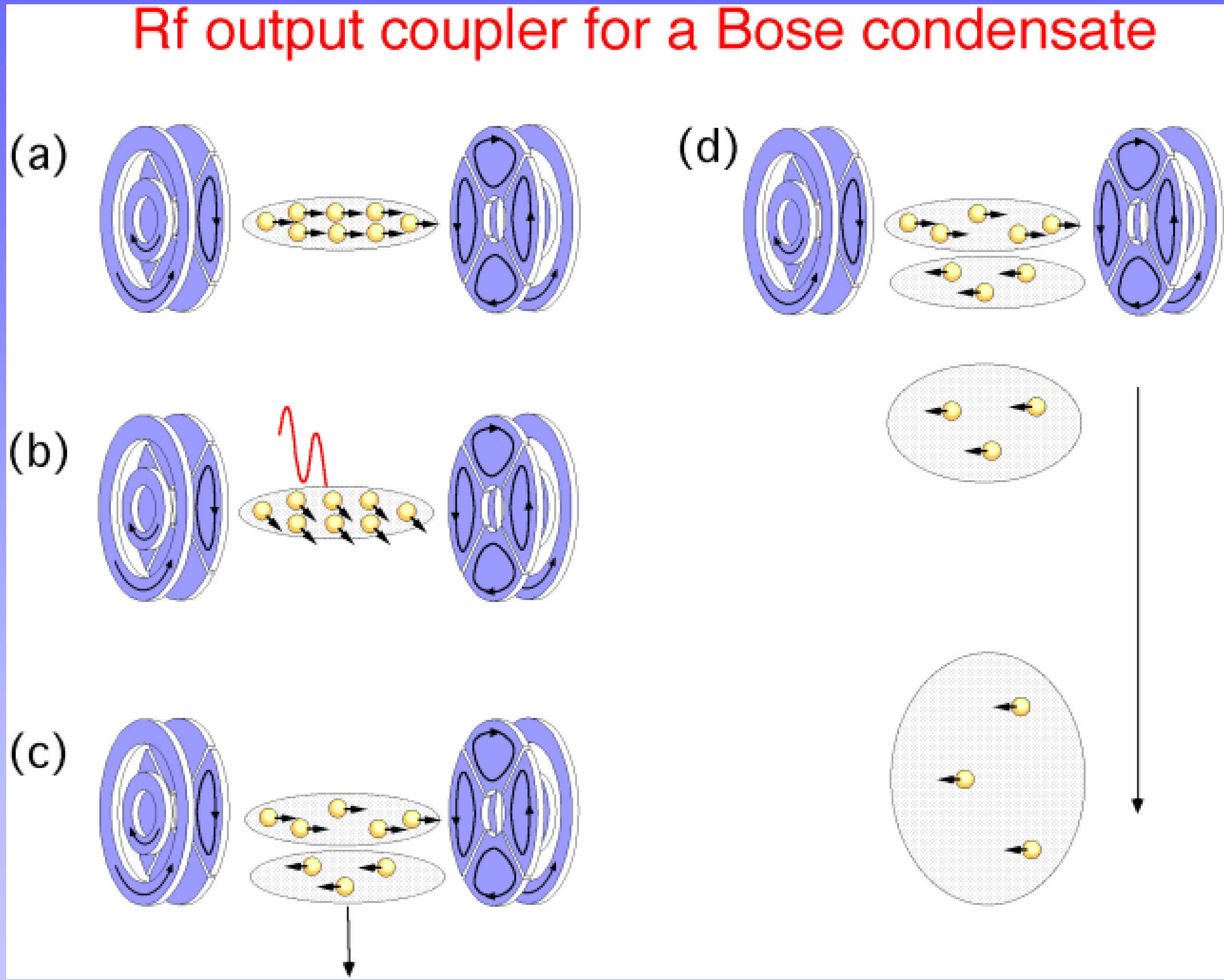
$$\Lambda_{dB} = \frac{h}{m\,\Delta v} = \frac{h\,\Delta t}{m\,d}$$



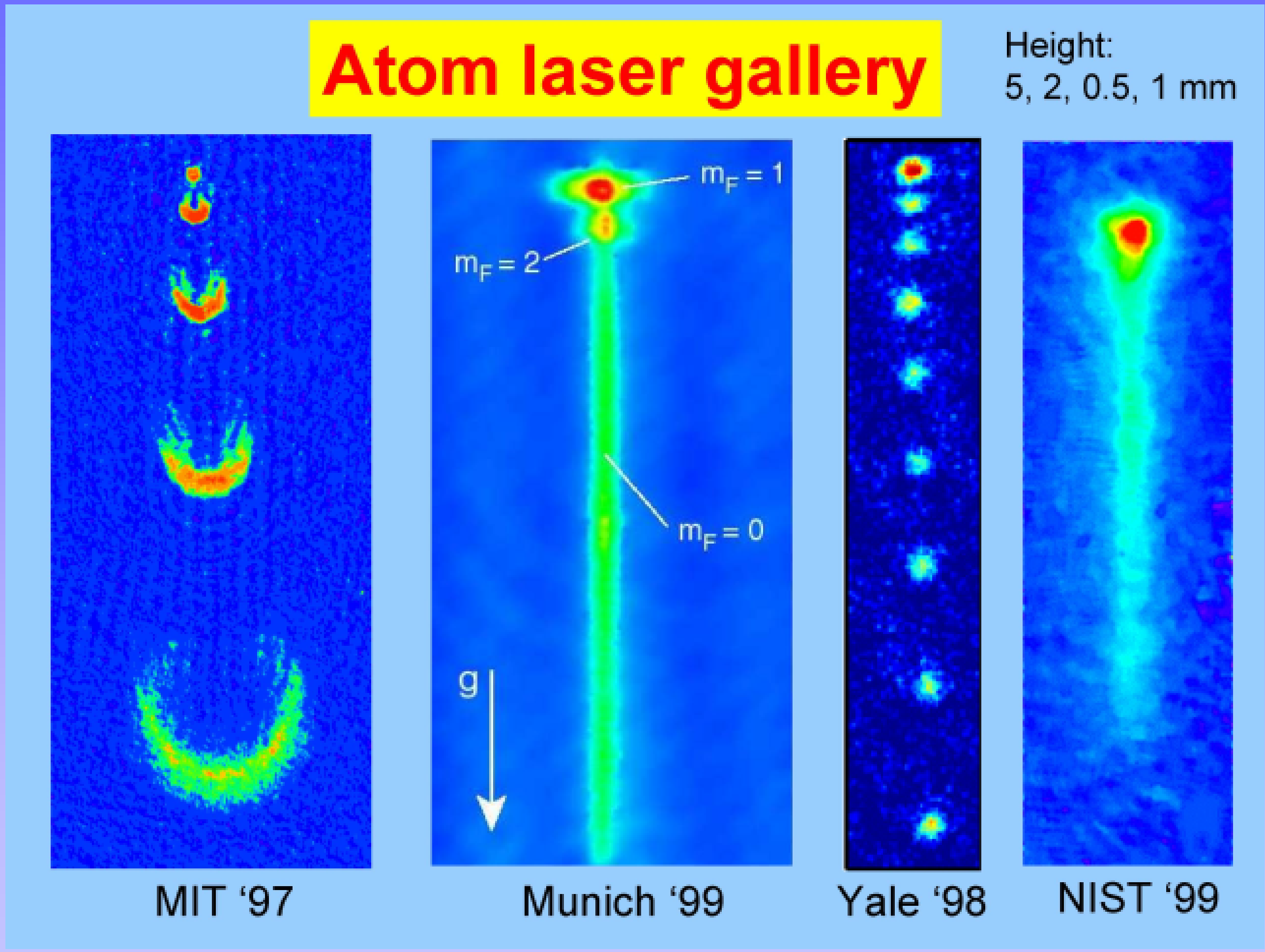
W. Ketterle *et al.* @ MIT

Output Coupler for a Bose condensate



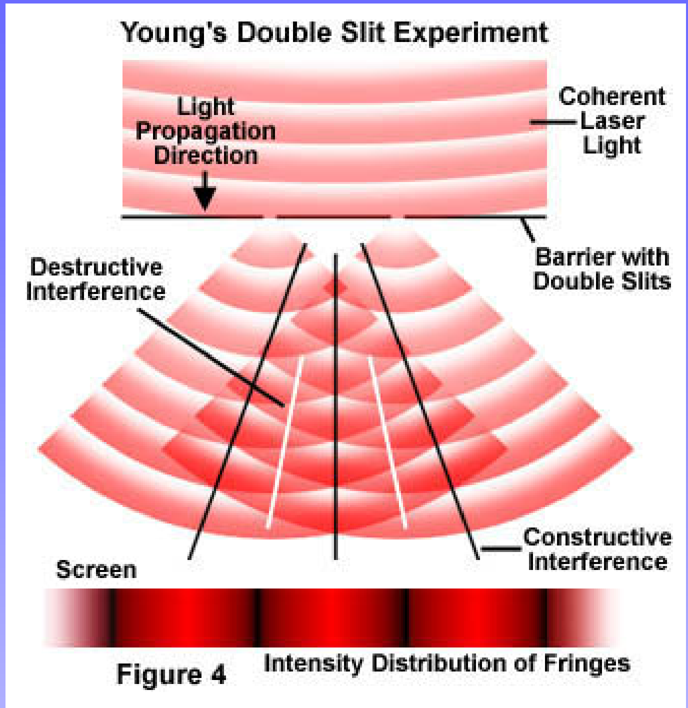


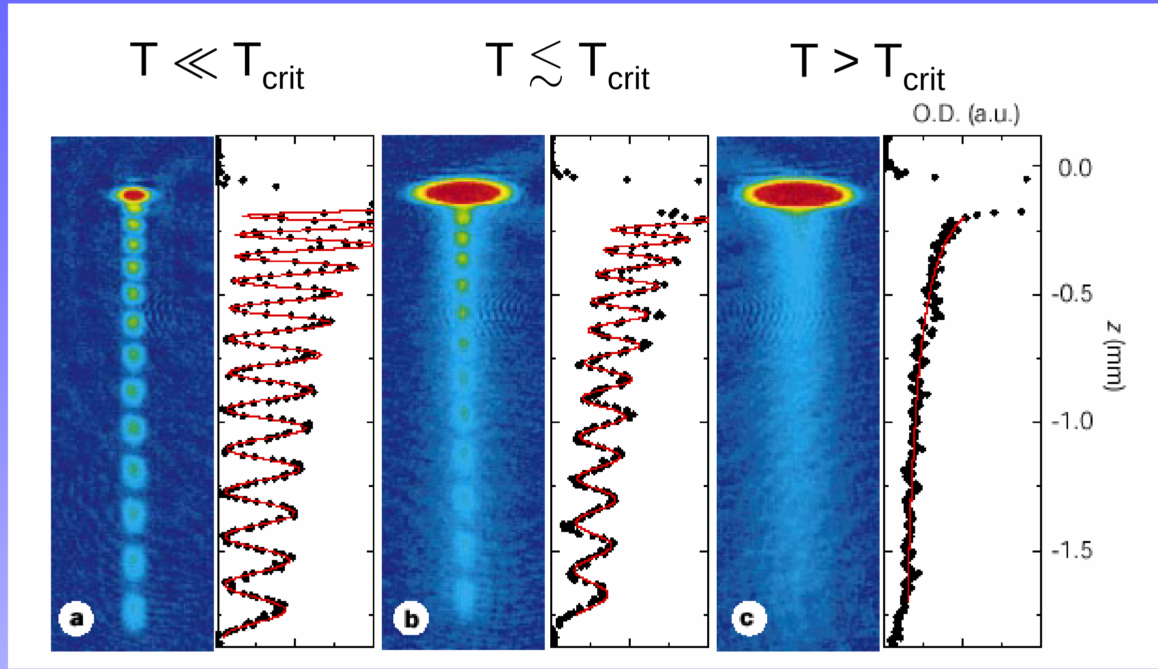
Atom Lasers

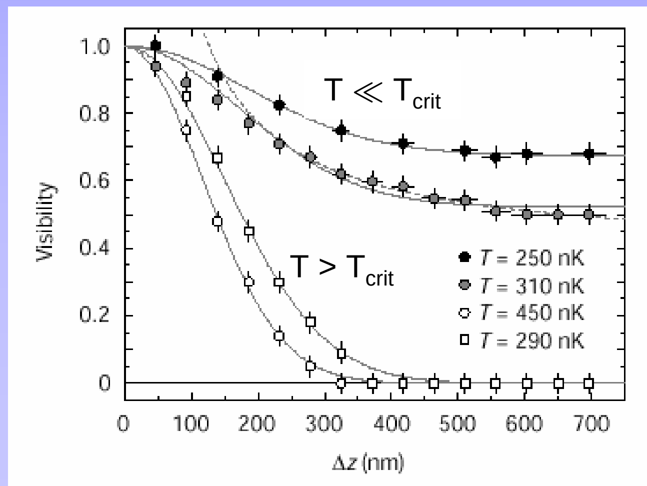


Atom laser gallery
Height:
5, 2, 0.5, 1 mm
m_F = 1
m_F = 2
m_F = 0
g
MIT '97
Munich '99
Yale '98
NIST '99

Coherence of the atom laser







T. W. Hänsch *et al.* @ Universität München

Gross-Pitaevskii equation

Gross-Pitaevskii equation for BEC mean-field interaction

$$i\hbar \frac{d\psi}{dt} = -\frac{\hbar^2}{2m}\nabla^2\psi + U(r)\psi + g\,|\psi|^2\,\psi$$

A < 0 attraction
A = 0 ideal gas
A > 0 repulsion

$$g = \frac{4\pi\,\hbar^2 A}{m}$$

ensemble creates „mean field“ (prop. to number density), which gives additional „potential“ in Schrödinger equation

$\rightarrow$ simple „mean-field theory“ with nonlinear behavior many phenomena (expansion, sound, collective oscillations) can be understood in this way !!

Thomas-Fermi regime

stationary solution **large ensemble**

Gross-Pitaevskii equation for BEC mean-field interaction

$$i\hbar \frac{d\psi}{dt} = -\frac{\hbar^2}{2m}\nabla^2\psi + U(r)\psi + g\,|\psi|^2\,\psi$$

A < 0 attraction
A = 0 ideal gas
A > 0 repulsion

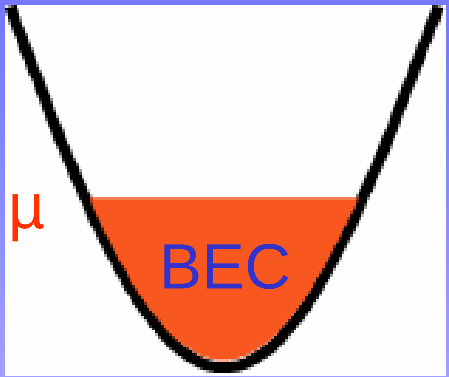
$$g = \frac{4\pi\,\hbar^2 A}{m}$$

very simple solution

$$n(\vec{r}) \propto \mu - U(\vec{r})$$

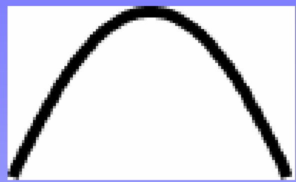
Thomas-Fermi limit

U(r)



+

n(r)~μ-U(r)



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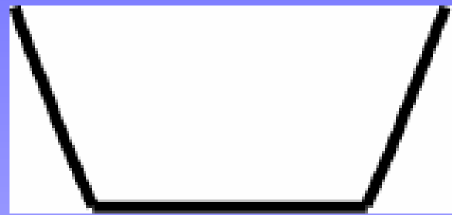


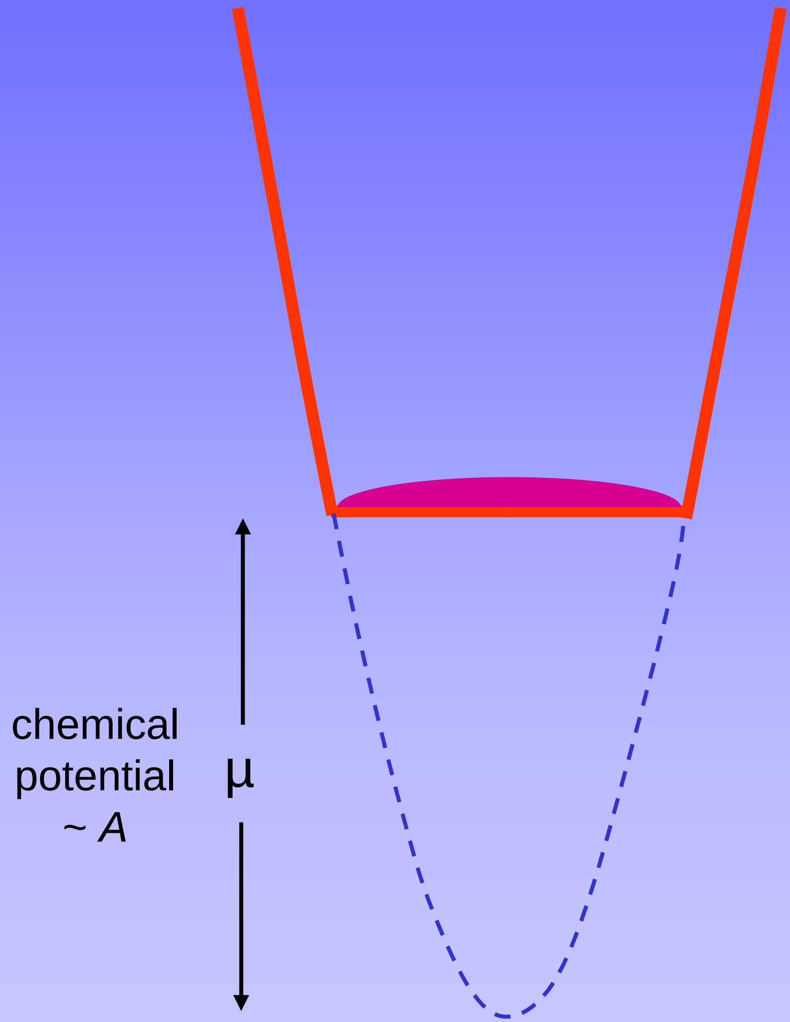
harmonic potential

mean-field potential

total potential

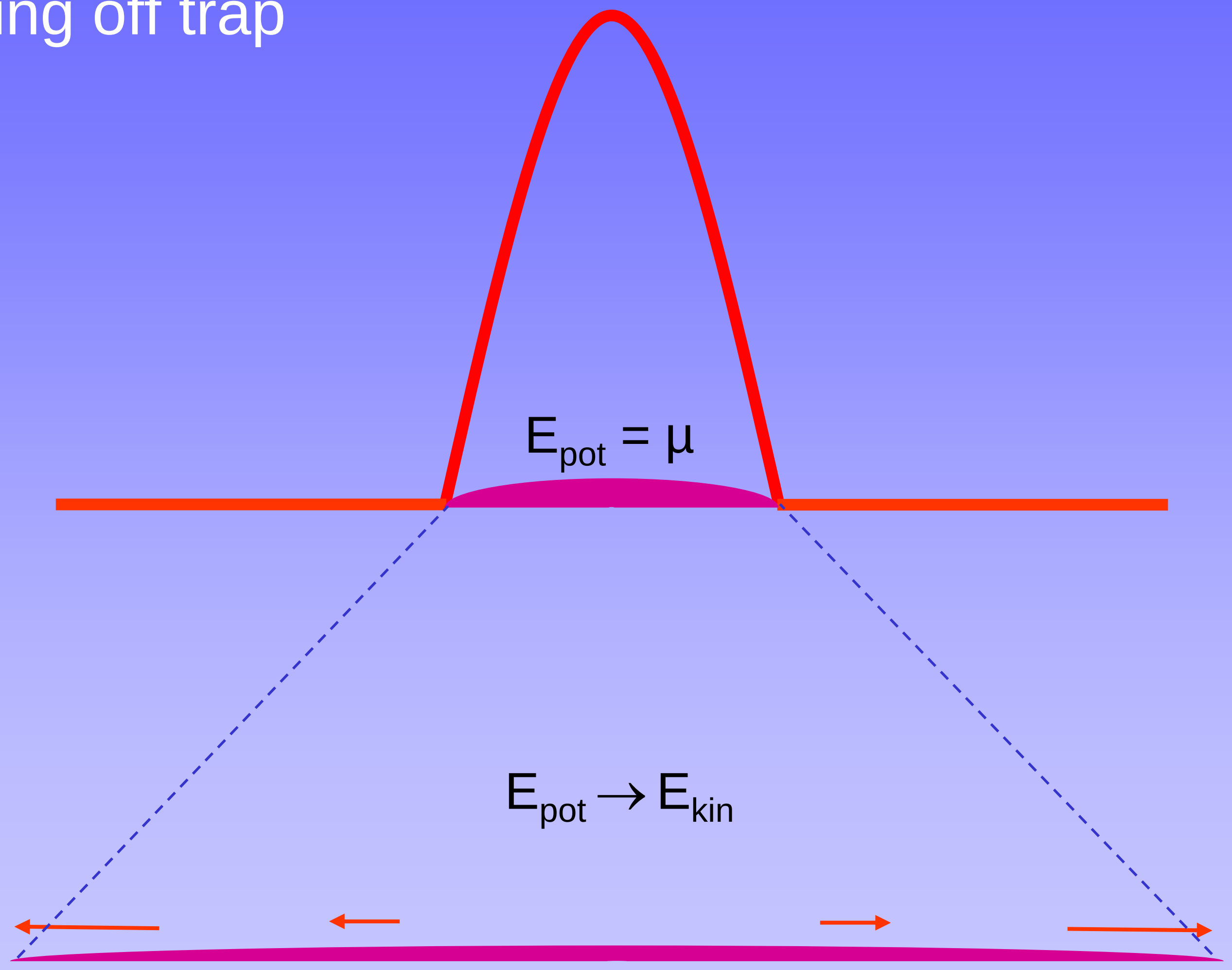
BEC density distribution is inverse shape of trap potential

Effect of interactions



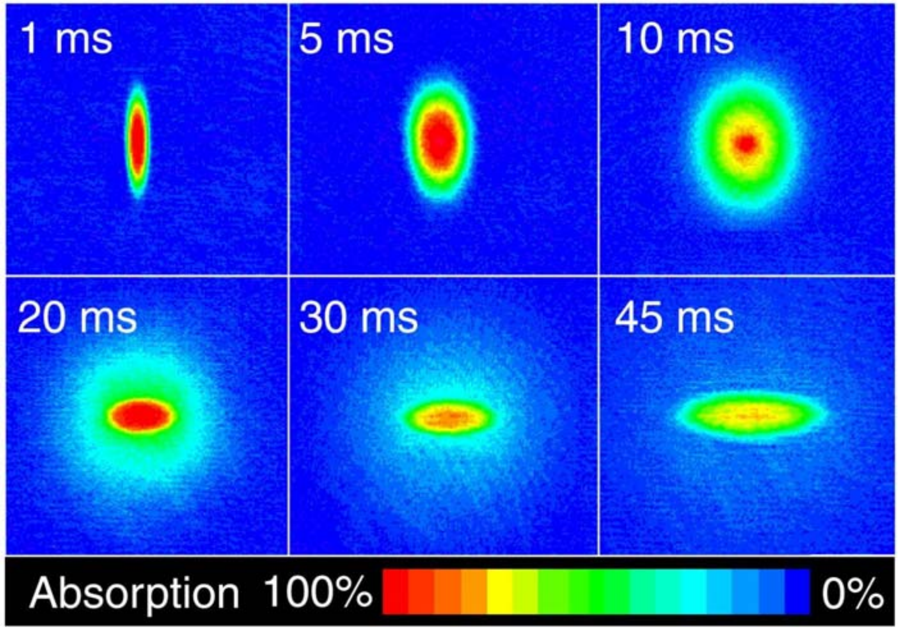
Expansion

switching off trap

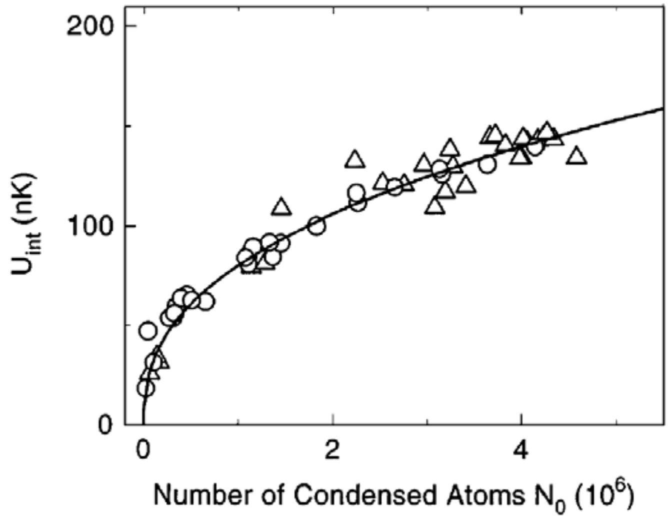


Influence of interactions

Free expansion of a Bose condensate



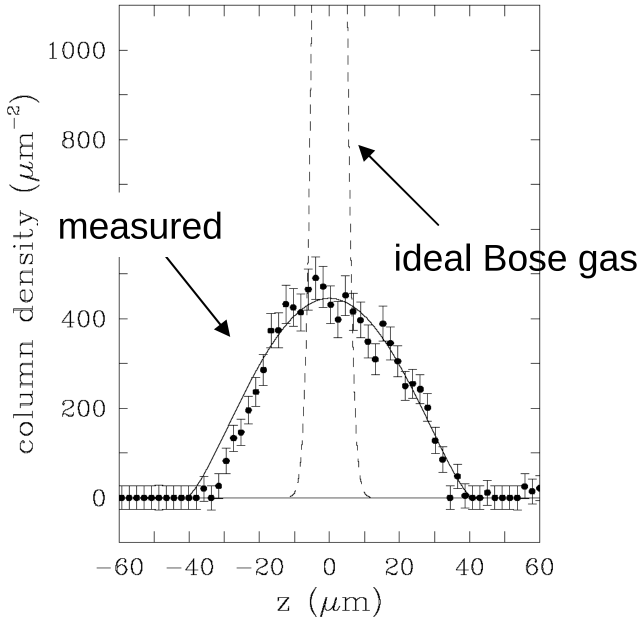
Internal energy



W. Ketterle *et al.* @ MIT

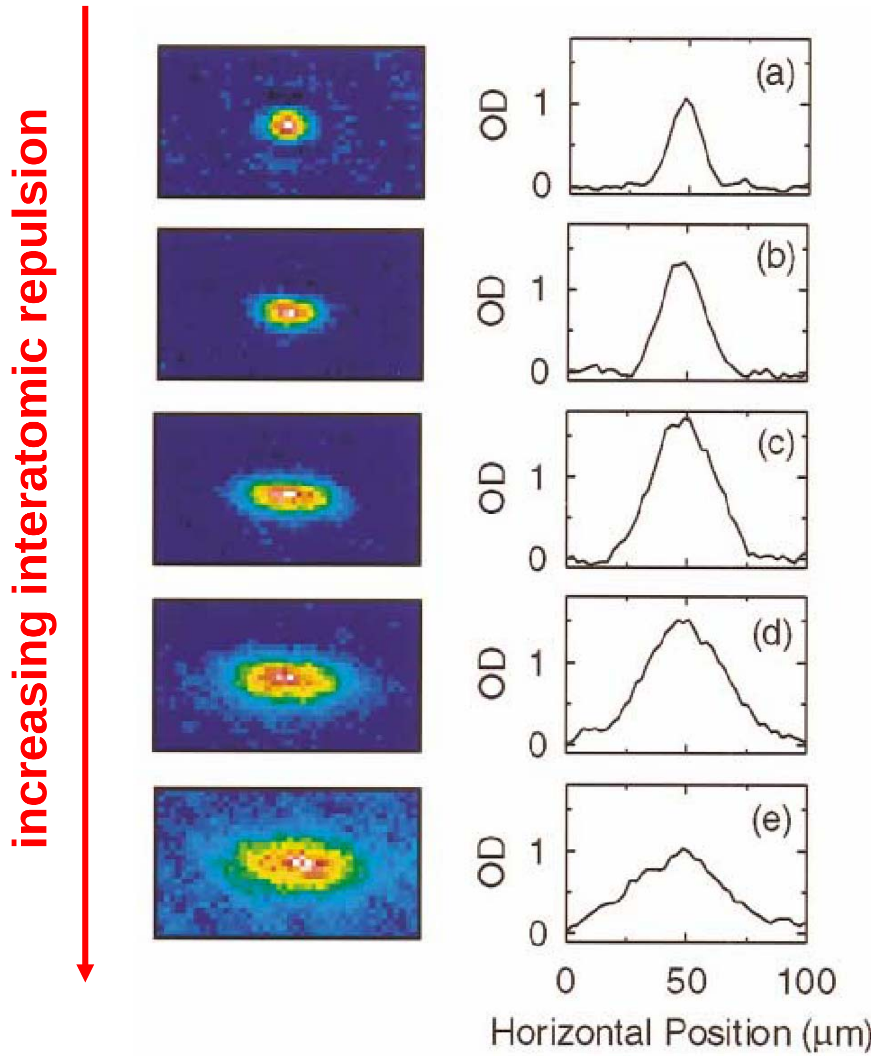
Influence of interactions (cont’d)

Density distribution



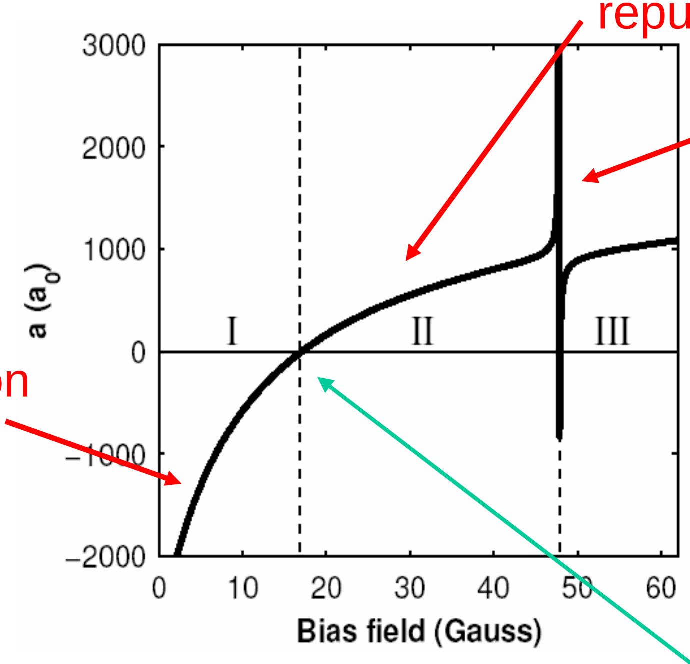
L. Hau *et al.* @ Harvard

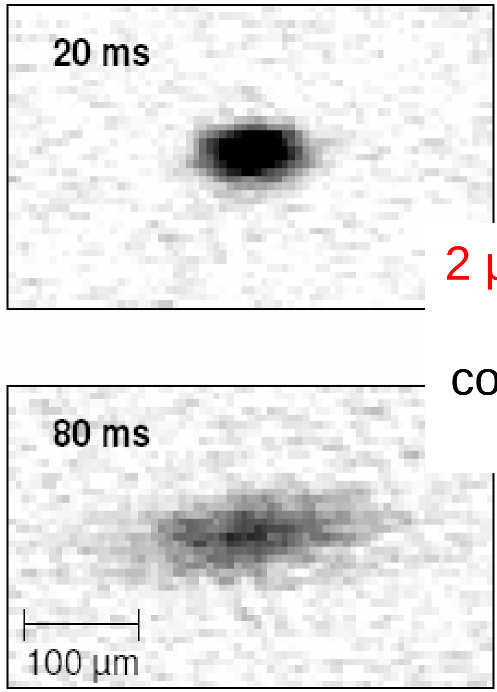
Manipulation of the interaction strength



E. Cornell, C. Wieman *et al.* @ JILA Boulder

Changing the mean-field interaction





2 µm in 100 ms
= 7 cm/h
corresponds to
50 pK !!!

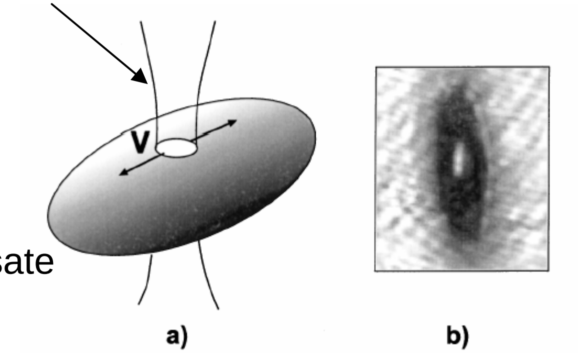
courtesy Rudi Grimm (Universität Innsbruck)

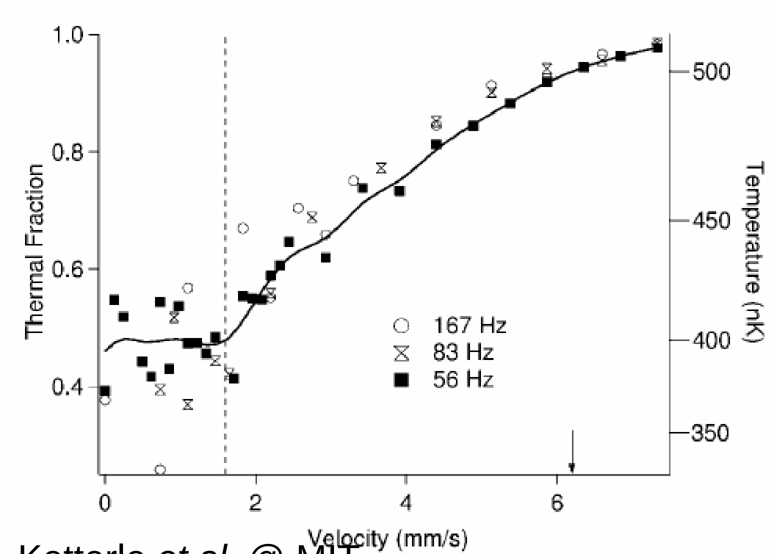
Superfluidity

Critical velocity

stirring laser beam

Bose condensate



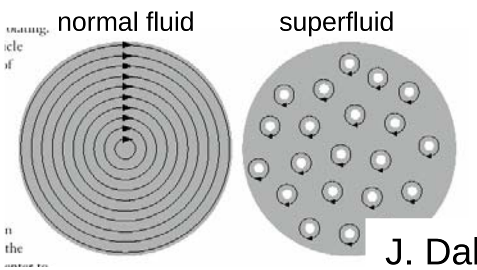


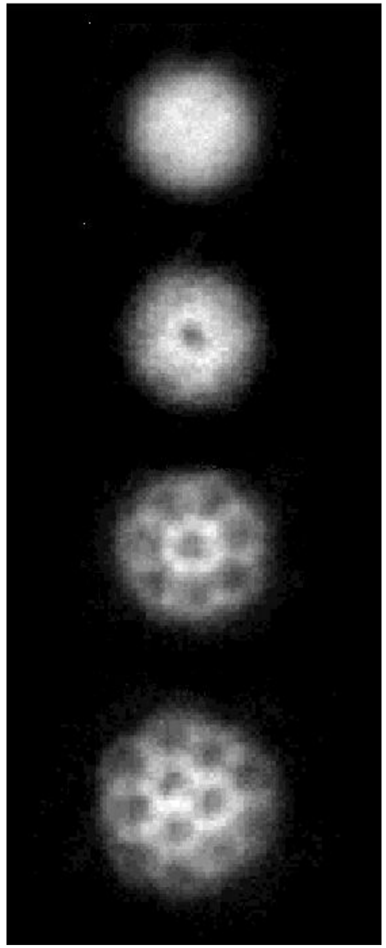
W. Ketterle *et al.* @ MIT

Quantized flux vortices

quantization of the velocity field due to macroscopic wavefunction

$$|\mathbf{v}| = n\,\frac{h}{m}$$

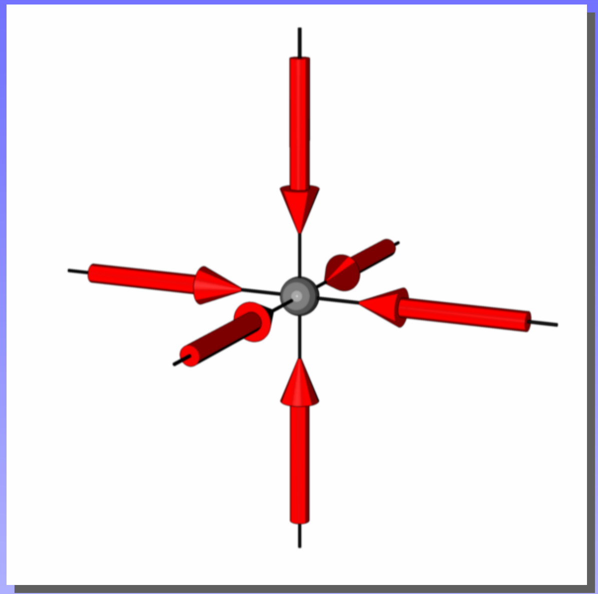


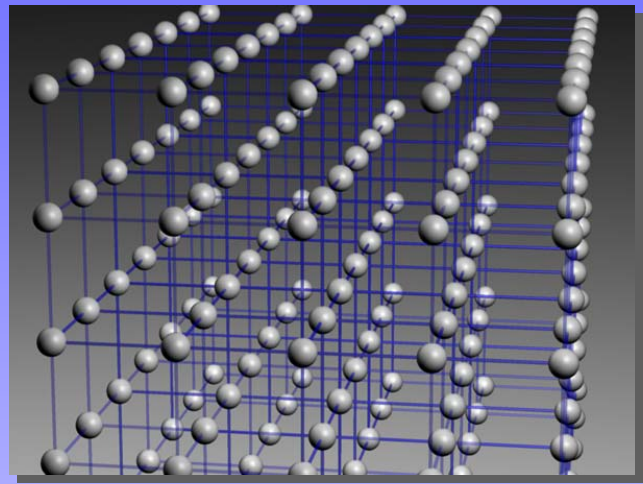


J. Dalibard *et al.* @ ENS, Paris

BEC in 3D Lattice Potential

courtesy Immanuel Bloch (Universität Mainz)





- ***Resulting potential consists of a simple cubic lattice***
- ***BEC coherently populates more than 100,000 lattice sites***

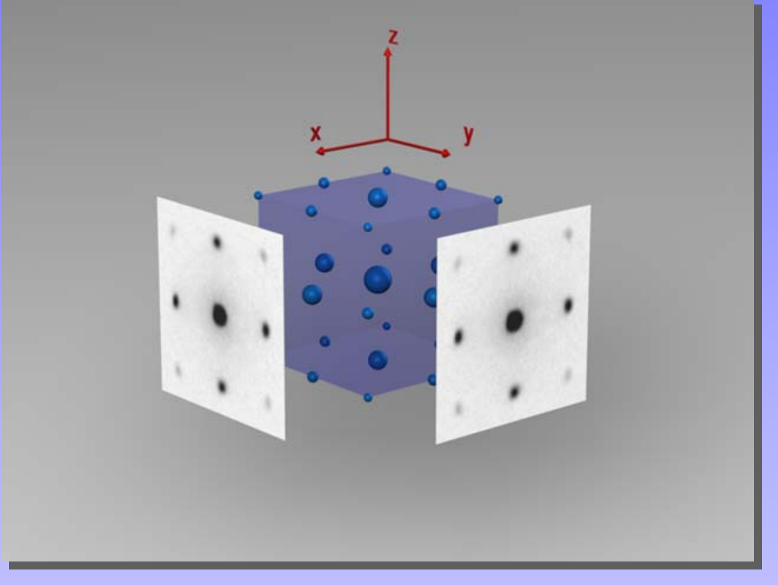
V_0 ***up to 22*** E_{recoil}

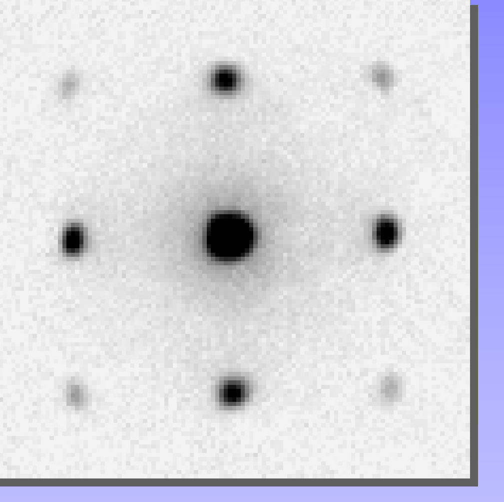
ω_r ***up to*** $2\pi \times$ ***30 kHz***

n ≈ ***1-5 atoms on average per site***

Interference Pattern of matter waves

courtesy Immanuel Bloch (Universität Mainz)

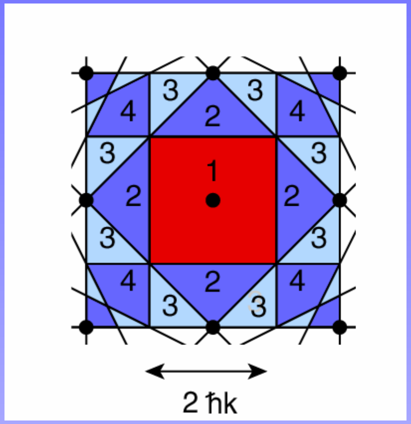




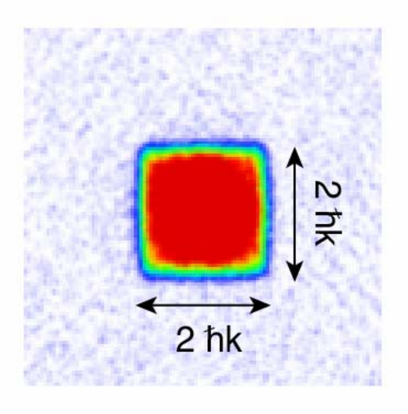
Mapping Brillouin zones

courtesy Immanuel Bloch (Universität Mainz)

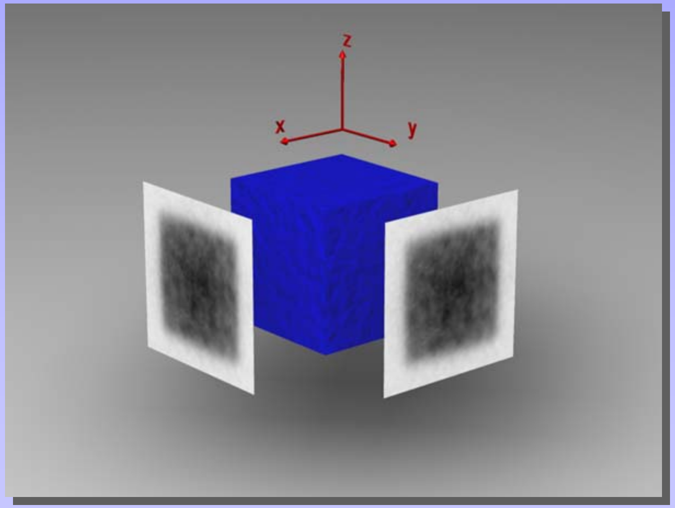
Brillouin Zones in 2D



Momentum distribution of a dephased condensate after turning off the lattice potential adiabtically



2D



3D

M. Greiner *et al.*, PRL **87**, 160405 (2001)

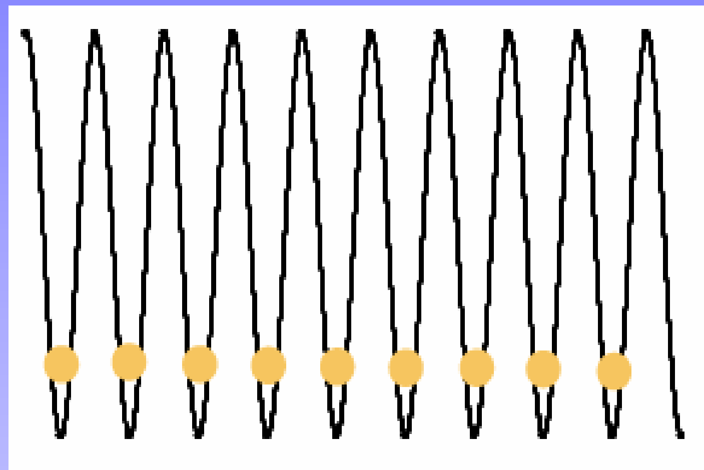
Basic idea of a Mott insulator

two different quantum phases (*T*=0)
separated by a ***quantum phase transition***

BEC (superfluid)

- strong tunnel coupling
- fixed phase relations
- fluctuations of site occupation numbers

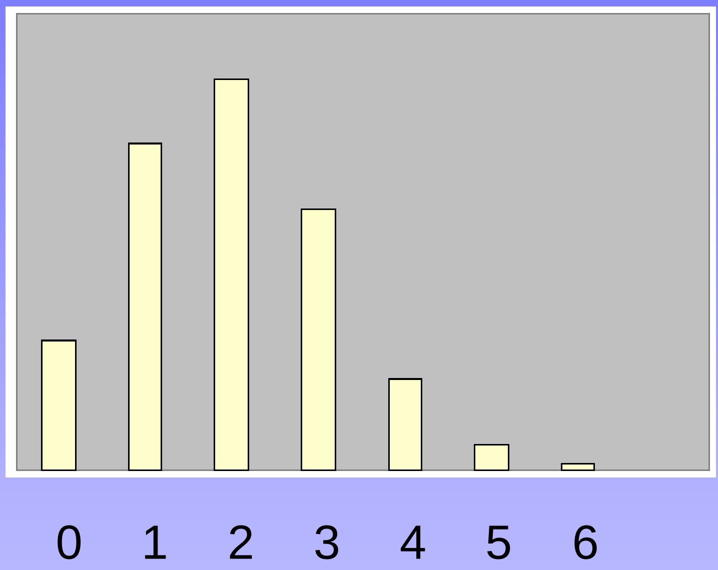
Mott insulator



- weak tunnel coupling
- no phase relations
- no fluctuations of site occupation numbers

Number fluctuation per lattice site

superfluid

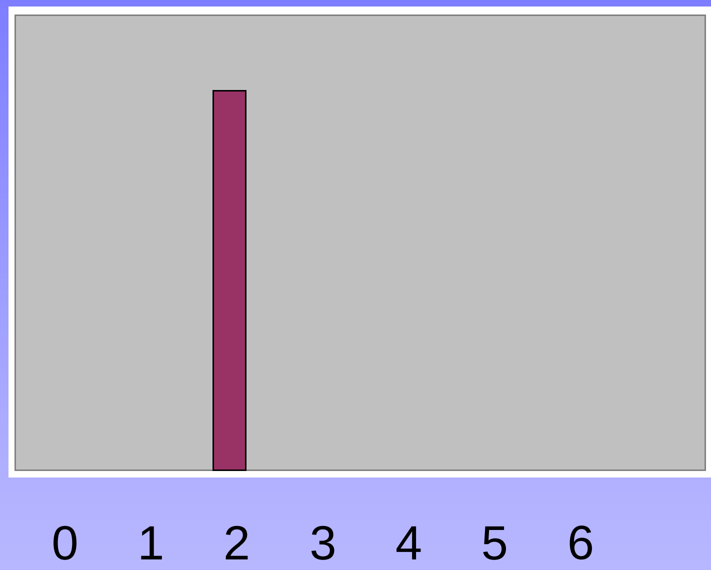


coherent state

(interference possible)

<n> =2

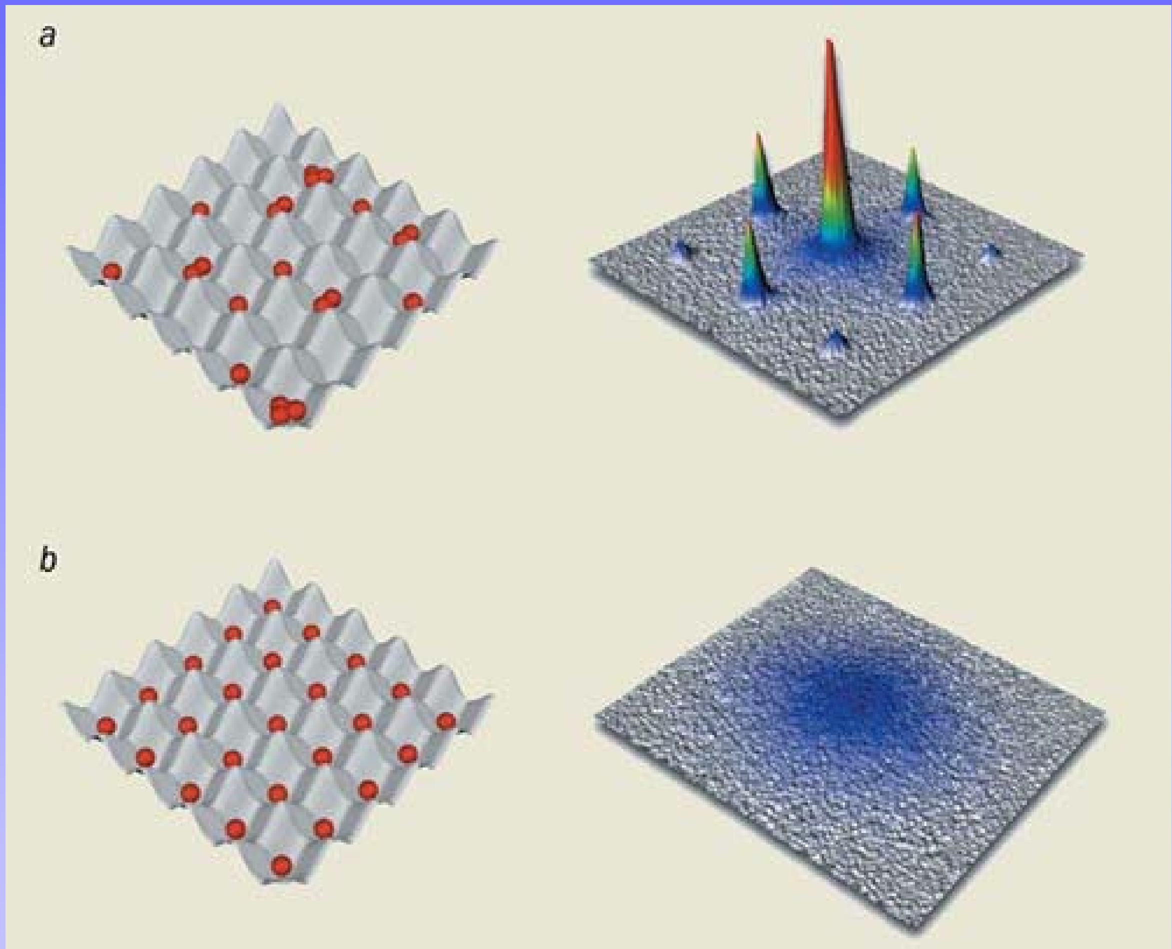
Mott insulator



number state

(no interference)

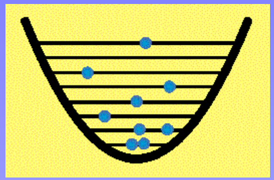
Observation of the Mott insulator

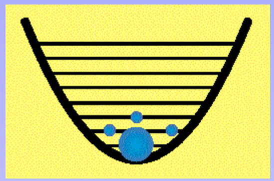


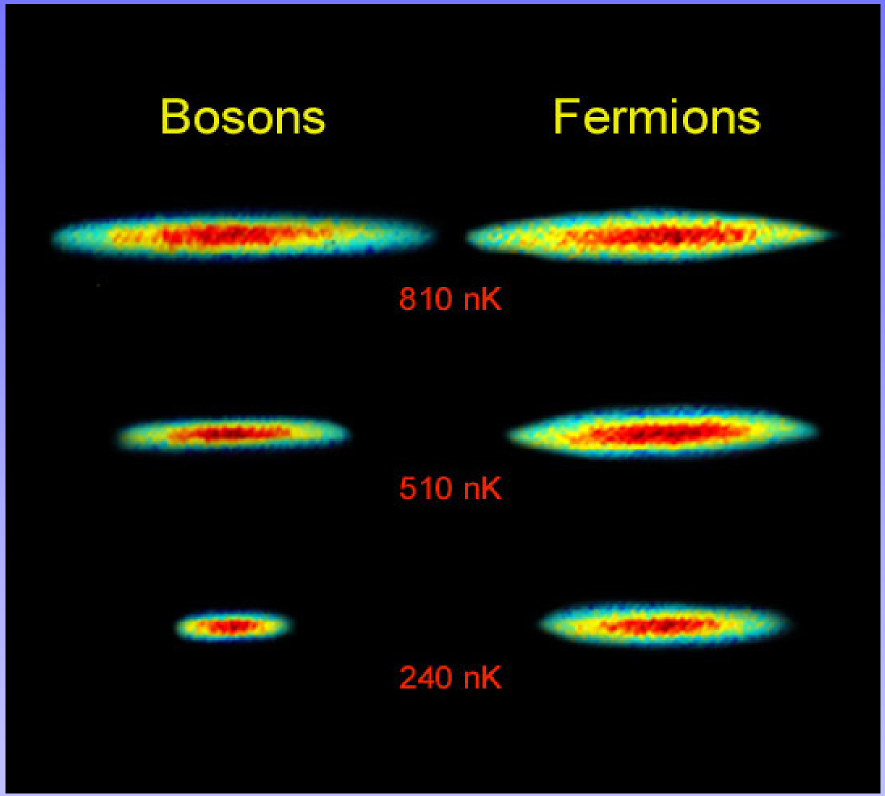
M. Greiner et al., Nature **415**, 39 (2002)
I. Bloch, Physics World, April 2004

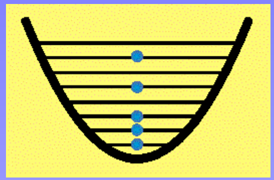
Degenerate Fermi gases

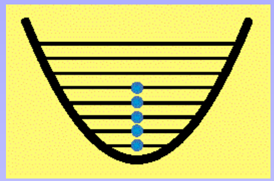
Atomic white dwarf (Pauli pressure)











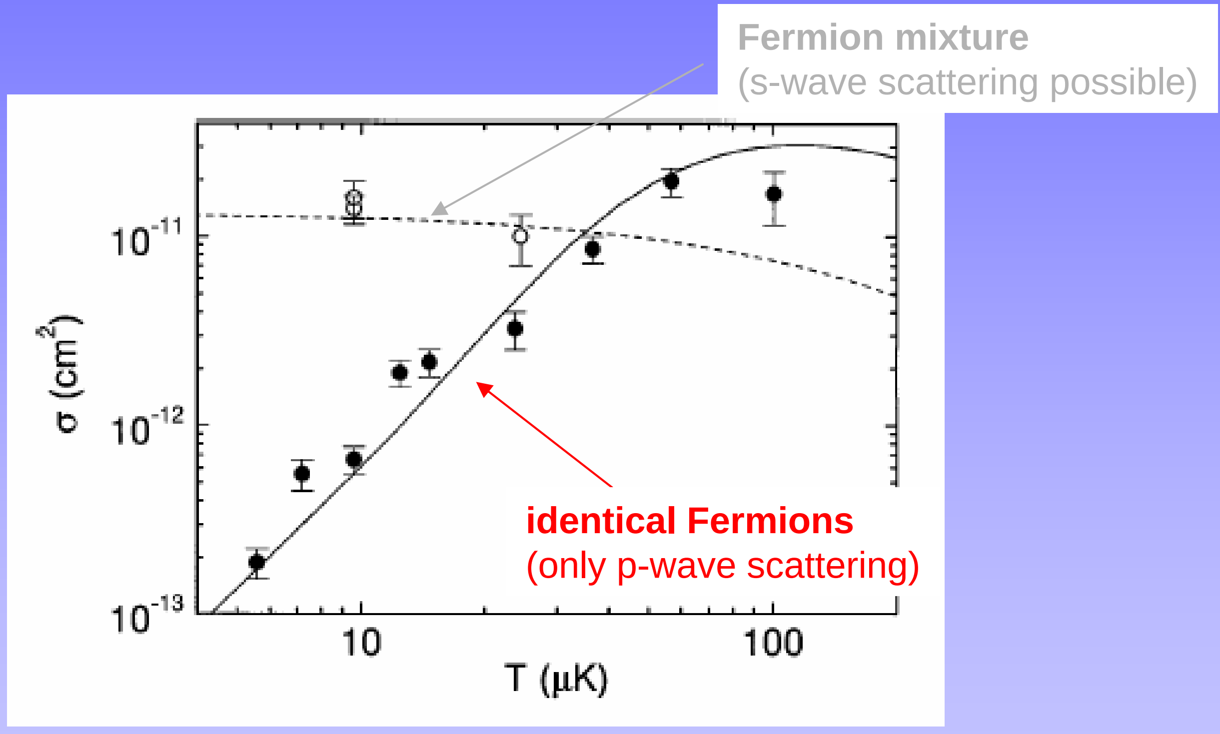
R. Hulet *et al.* @ Rice University, Houston

Fermions

Suppression of elastic collisions

identical Fermions:

s-wave scattering length A = 0



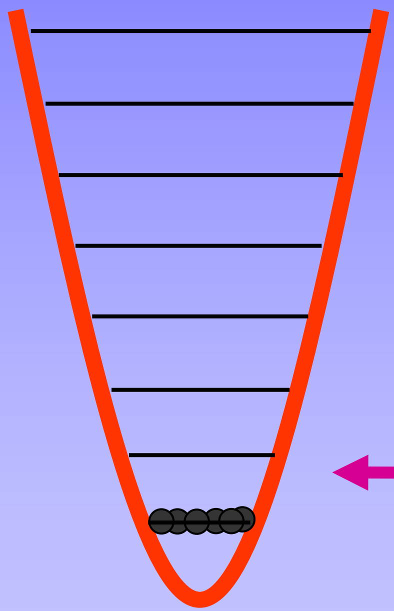
B. deMarco *et al.*, PRL **82**, 4208 (1999)

Two classes

courtesy Rudi Grimm (Universitäat Innsbruck)

Bosons

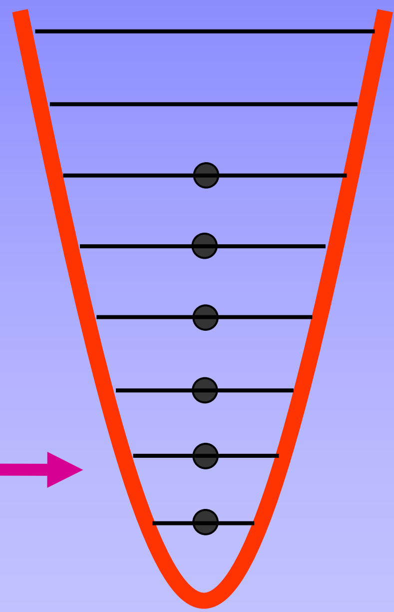
integer spin



all in ground state:
Bose-Einstein condensate

Fermions

half-integer spin



only one particle per state:
degenerate Fermi gas

trapped atoms at T=0

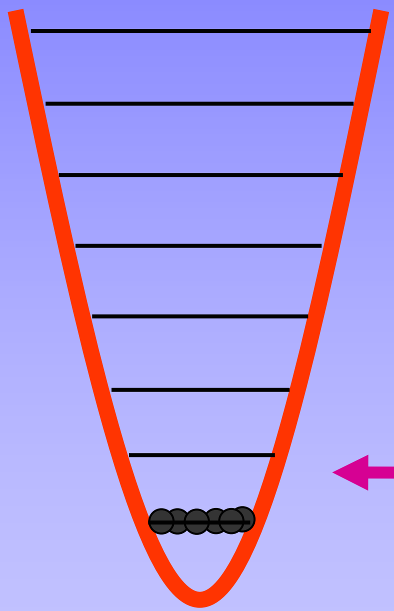
these two worlds are connected !

Two classes

courtesy Rudi Grimm (Universitäat Innsbruck)

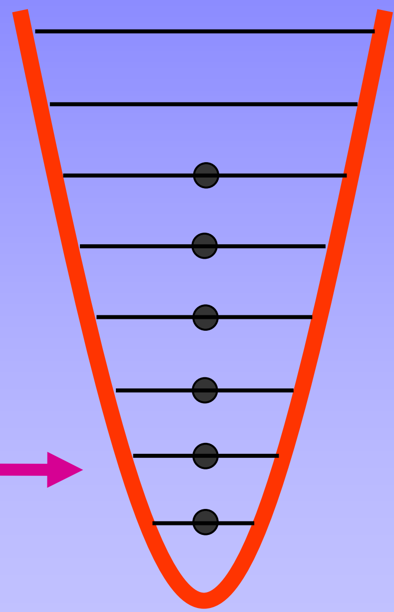
Bosons
integer spin

Fermions
half-integer spin



trapped atoms at *T*=0

„pairing“ is the key



all in ground state: Bose-Einstein condensate

only one particle per state: degenerate Fermi gas

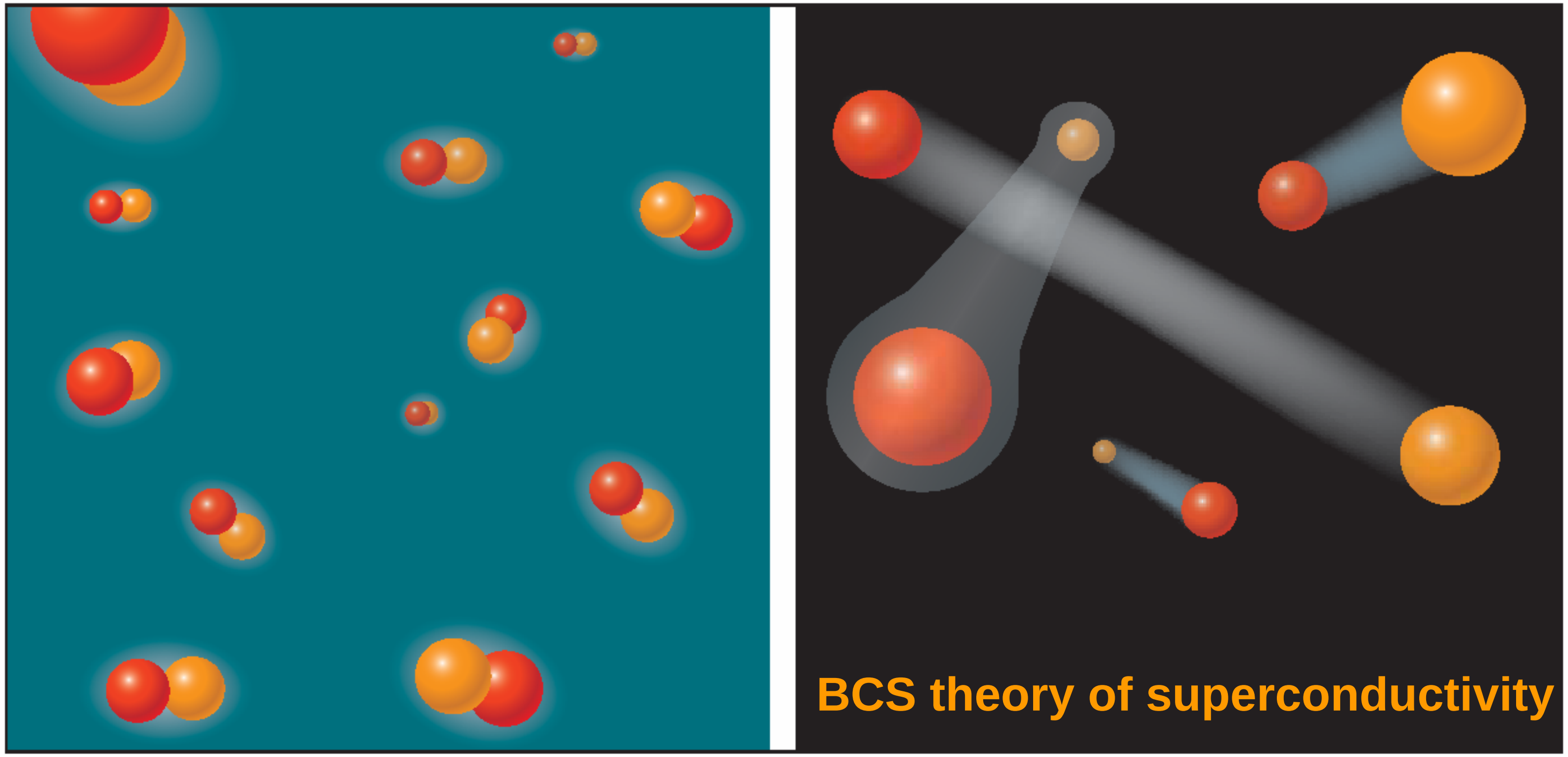
BEC-BCS crossover

courtesy Rudi Grimm (Universitäat Innsbruck)

crossover

molecules

Cooper pairs

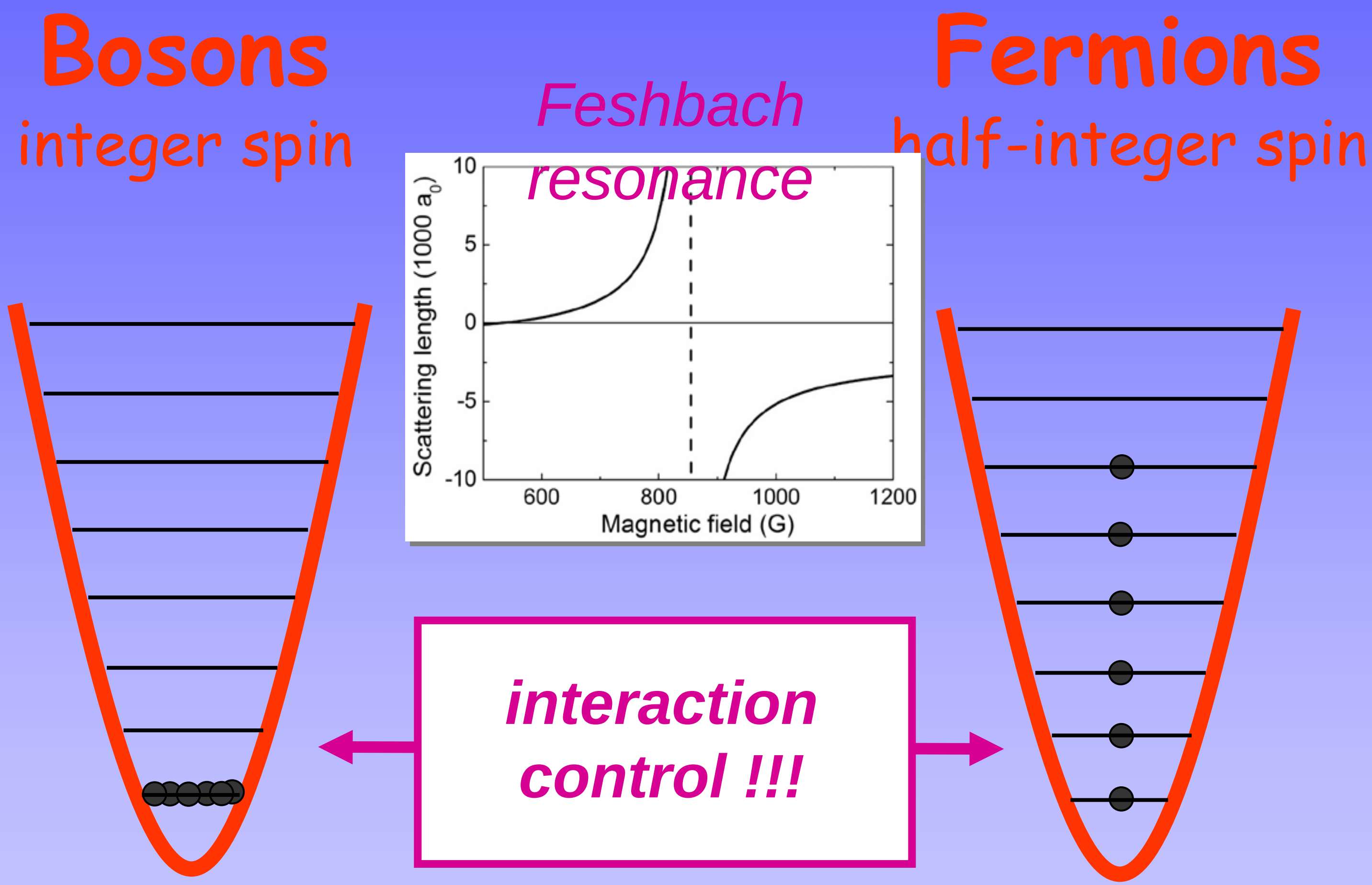


Tango or twist? In a magnetic field, atoms in different spin states can form molecules (*left*). Vary the field, and they might also form loose-knit Cooper pairs.

A. Cho, Science **301**, 750 (2003)

Two classes

courtesy Rudi Grimm (Universitäat Innsbruck)



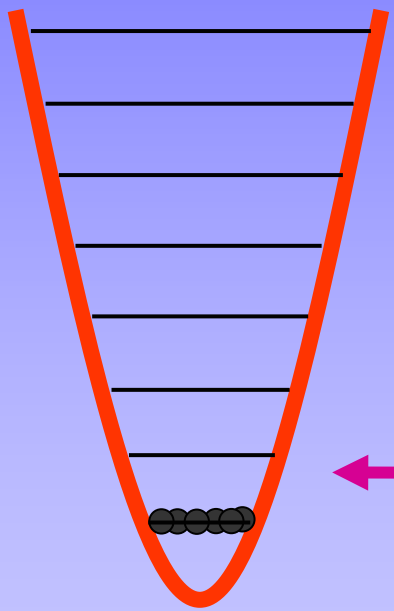
all in ground state: Bose-Einstein condensate

only one particle per state: degenerate Fermi gas

Two classes

courtesy Rudi Grimm (Universitäat Innsbruck)

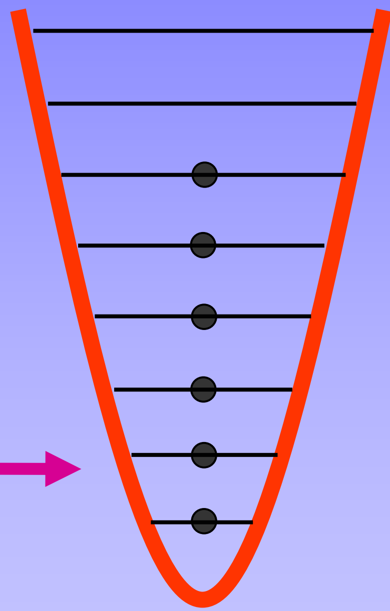
Bosons
integer spin



neutron star

exotic states of matter

Fermions
half-integer spin



all in ground state: Bose-Einstein condensate

only one particle per state: degenerate Fermi gas

Two classes

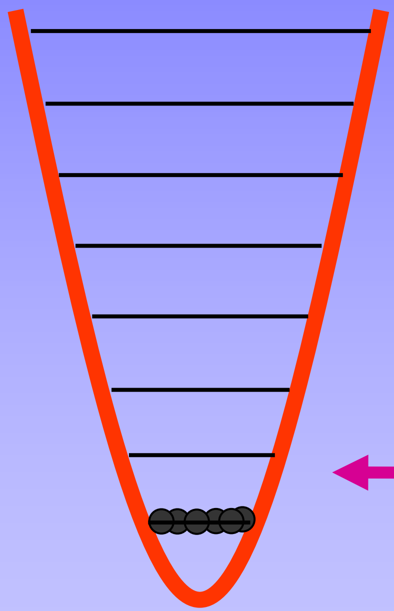
courtesy Rudi Grimm (Universitäat Innsbruck)

Bosons

integer spin

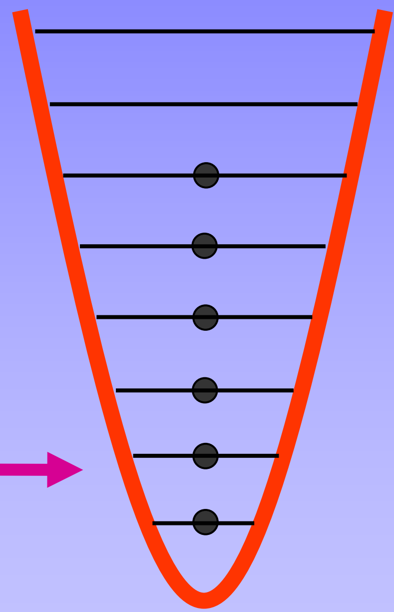
Fermions

half-integer spin



model system for high-Tc superconductors

superfluidity ?

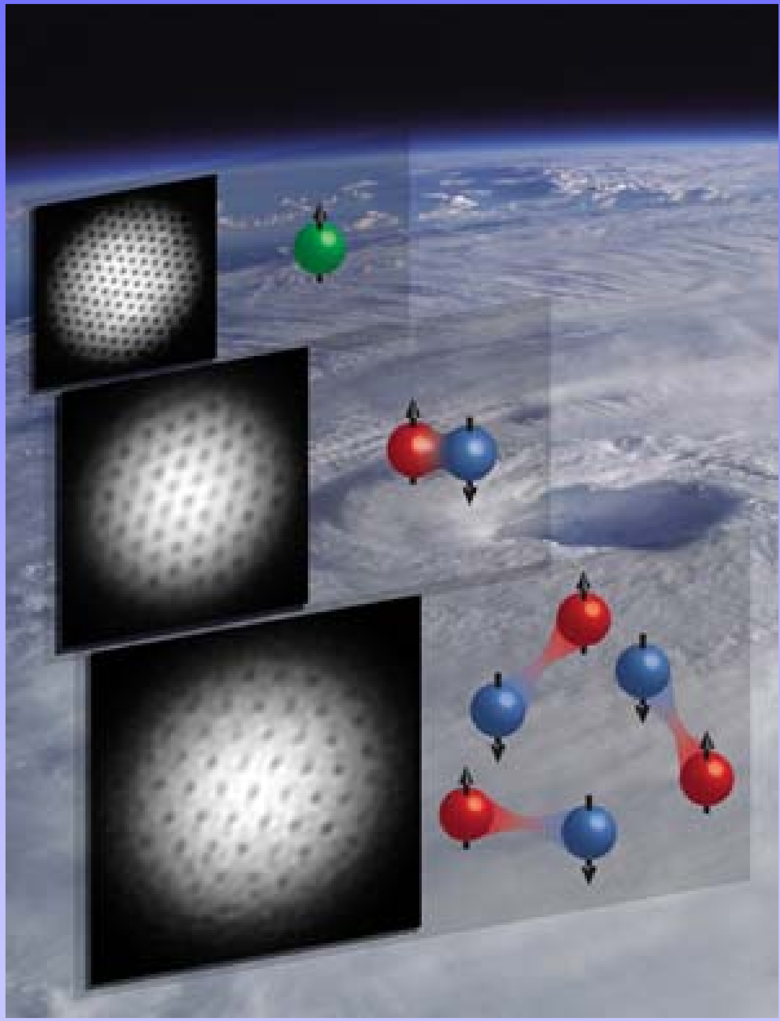


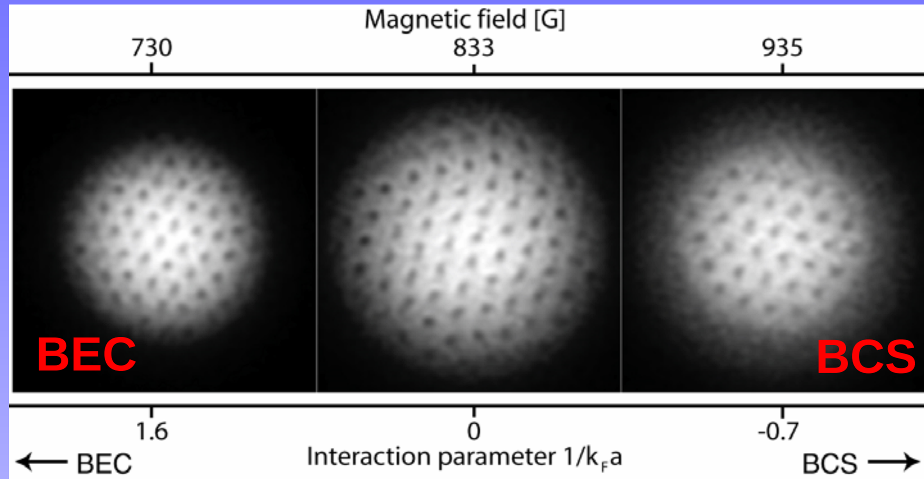
all in ground state:
Bose-Einstein condensate

only one particle per state:
degenerate Fermi gas

Superfluidity in a paired Fermi gas

Vortices in the BEC and BCS phase





M.W. Zwierlein *et al.,* Nature **435**, 1047 (2005)

Summary of Lecture 2

- **Quantum statistics of Bosons and Fermions**
  - quantum statistics of composite particles
  - Bose-Einstein condensation and Fermi degeneracy

- **Bose-Einstein condensation**
  - giant matter-wave interference
  - role of elastic collisions (mean-field interaction, superfluidity)
  - Bose condensates in optical lattices (diffraction, Mott insulator)

- **Degenerate Fermi gases**
  - suppression of s-wave scattering
  - Pauli pressure
  - BEC-BCS crossover